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CONTRACT REPORT ARBRL-CR-00432

A MODEL OF THE TRAVELING CHARGE

Prepared by

Paul Gough Associates, Inc.
Portsmouth, NH 03801

July 1980

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER CONTRACT REPORT ARBRL-CR-00432	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE *(and Subtitle)* A Model of the Traveling Charge		5. TYPE OF REPORT & PERIOD COVERED Contractor 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR*(s)* P. S. Gough		8. CONTRACT OR GRANT NUMBER*(s)* DAAK11-79-C-0071
9. PERFORMING ORGANIZATION NAME AND ADDRESS Paul Gough Associates, Inc. Portsmouth, NH 03801		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Armament Research & Development Command US Army Ballistic Research Laboratory (ATTN: DRDAR-BL) Aberdeen Proving Ground, MD 21005		12. REPORT DATE July 1980 13. NUMBER OF PAGES 150
14. MONITORING AGENCY NAME & ADDRESS*(if different from Controlling Office)*		15. SECURITY CLASS. *(of this report)* Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT *(of this Report)*

Distribution limited to US Government agencies only; Test and Evaluation; Jul 80. Other requests for this document must be referred to Director, USA Ballistic Research Laboratory, ATTN: DRDAR-TSB, Aberdeen Proving Ground, MD 21005.

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

Traveling Charge Gun
1-D Gas Dynamic Models
Ultra-High Burning Rate Propellant
High Velocity Guns

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

A model is described to permit the theoretical evaluation of the ballistic performance of an end-burning traveling charge. The column of gas between the breech and the regressing propellant surface is analyzed as a one-dimensional, unsteady, inviscid flow with heat loss to the wall. The solid propellant may be represented either as rigid or as an unsteady, one-dimensional, viscoplastic continuum which interacts with the tube wall through the action of friction.

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To enable the assessment of potential or ideal performance, as well as that currently available from measured rates of burning, the combustion of the propellant may be prescribed according to any of several ideal criteria. The reactants may be required to come to rest, to attain a predetermined Mach number or to induce a predetermined pressure on the unreacted side of the interface or a predetermined acceleration of the projectile. While there are strong arguments to rule out the possibility of a steady combustion wave with supersonic reactants, as viewed from the regressing surface, ideal models of such strong deflagration waves are provided so as to enable the assessment of the performance loss associated with a limitation to weak or subsonic deflagrations.

The breech may be taken to be gas-permeable and the solution can be continued past muzzle exit in order to study blowdown of the tube. The solution is obtained by means of an explicit two-level finite difference scheme and uses the method of characteristics at the external boundaries and at the interface between the gas and the solid propellant.

Sample calculations are provided in order to demonstrate the stability of the method of solution and to benchmark its accuracy. Comparison with a case which admits an exact solution is provided and studies are made of mesh indifference and of the global conservation of mass and energy.

Foreword

Technical cognizance for the subject contract
has been provided by Mr. P. G. Baer
U.S. Army Ballistic Research Laboratory DRDAR-BLP

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1.0 INTRODUCTION

This report is concerned with the interior ballistic performance of a particular type of gun propelling charge which is attached to the base of the projectile and which burns in successive planar layers, combustion beginning at the rear face of the charge and progressing towards the projectile base. A propelling charge of this type is referred to as an end-burning traveling charge and is of current interest as a potential solution in applications requiring very high muzzle velocities--of the order of 3 km/sec.

In order to permit a theoretical evaluation of the performance to be expected of such a charge, we have developed a model of the one-dimensional continuum dynamics of the solid propellant and its products of combustion, the reaction zone being assumed sufficiently thin that it may be represented as an internal boundary condition. The purpose of the present report is to provide documentation of the model, including the mathematical formulation of the equations, the method of solution, and the structure and use of the computer program into which the model has been encoded.

Subsequent subsections of this introduction enlarge on the concept of the traveling charge, define the scope of the present effort, and summarize the modeling approach.

1.1 Background

Increasing the muzzle velocity offers two advantages: the time of flight of the projectile is reduced, increasing the likelihood of defeating a highly mobile target; and the terminal velocity is increased, at moderate range, increasing the likelihood of penetration of heavy armor. Present conventional weapons yield muzzle velocities of the order of 1.0-1.5 km/sec. Significant increases in effectiveness against the most mobile or most heavily armored targets would accrue if muzzle velocities as high as 3.0 km/sec could be obtained.

The relevance of the traveling charge concept to the design of weapons yielding such high velocities has been discussed in some detail in a recent review by May et al[1]. In the present report we confine our discussion to certain conceptual aspects of propelling charge performance in order to identify the theoretical factors which have motivated past and present interest in the traveling charge and to note certain questions which may be asked about the traveling charge concept; these questions motivate the present work.

Figure 1.1 illustrates, in a schematic form, the nature of the flows associated with a conventional granular propelling charge and an

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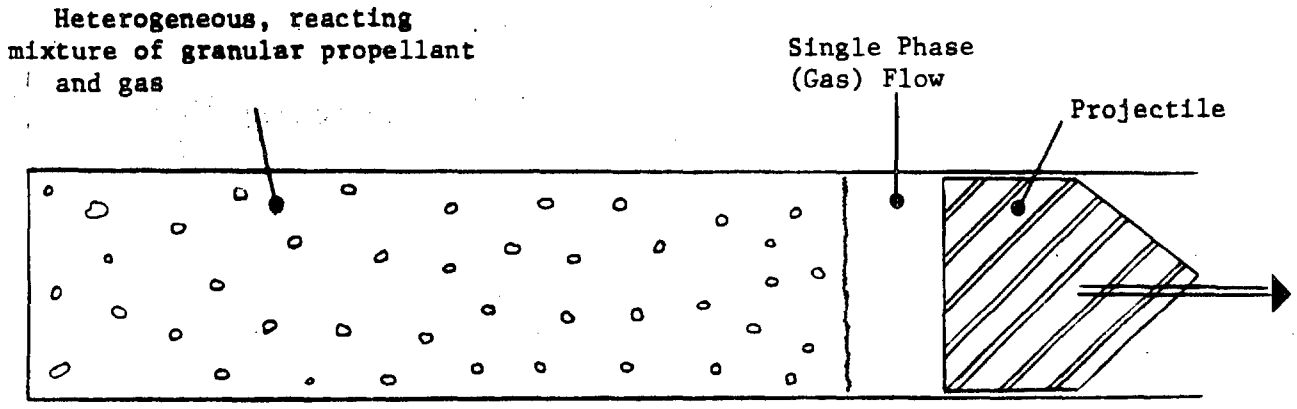
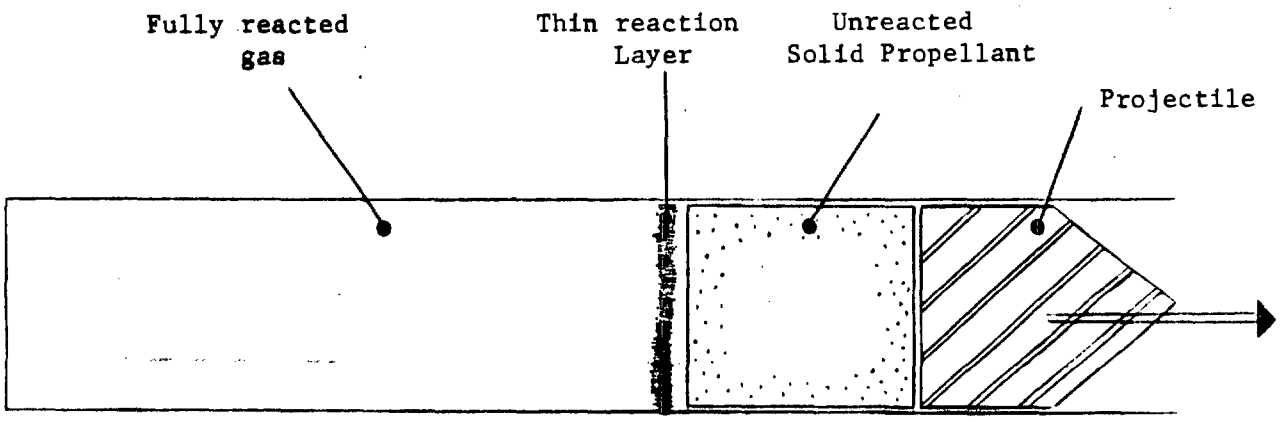
[1] *May, I. W., Baran, A. F., Baer, P. G. and Gough, P. S. "The Traveling Charge Effect" Proceedings of the 15th JANNAF Combustion Meeting 1978*



(a) Conventional granular propelling charge



(b) End-burning traveling charge

Figure 1.1 Schematic illustration of flows associated with conventional granular and end-burning traveling charges

end-burning traveling charge. In neither case do we discuss the details of the ignition process; details pertaining to the conventional charge may be found in the recent AIAA Progress Series volume edited by Krier and Summerfield[2].

Except for a relatively small region of single phase flow adjacent to the base of the projectile, the granular propellant tends to be found throughout the tube at all times prior to burnout. Therefore the flow is that of a heterogeneous reacting mixture in which the spacewise release of chemical energy is more or less uniform. The tendency of the mixture to become well stirred is due to the action of interphase drag. Conversely the presence of a region of single phase flow adjacent to the projectile base is due to the vanishing of the slip velocity at the base. The departure of the mixture from true well-stirredness can be pronounced in some cases, especially during ignition, and the ballistic consequences may be important[2]. However, by comparison with the end-burning traveling charge, in which the propellant and its products of combustion are separated by a thin layer in which the heat release occurs, the granular propelling charge looks very well-stirred indeed.

Taking the two-phase mixture to be well-stirred and having more or less uniform density leads to the classical one-dimensional solution of Lagrange[3] in which the velocity profile is linear and the pressure profile is parabolic. In particular, one may deduce that at any instant of time the base pressure p_{base} and the breech pressure p_{breech} are related according to the formula $p_{base} = p_{breech}/(1 + W/2)$ where W is the charge-to-projectile mass ratio. Also, one may show that the ratio of the kinetic energy of the propellant mixture to that of the projectile is $(W/3) : 1$.

These elementary classical results define the perspective from which the concept of the traveling charge appears to offer certain ballistic benefits as the charge-to-projectile mass ratio is increased. The relationship between p_{breech} and p_{base} describes the degree to which the pressure at the projectile base is reduced by the requirement that a gradient exist to accelerate the mixture. In conventional ammunition, with muzzle velocities of the order of 1 km/sec, W is approximately 0.50 so that $p_{base} \sim 0.80\ p_{breech}$. Now the propulsive capacity is clearly due to p_{base} while either the weapon weight is related to p_{breech} or, conversely, p_{breech} is limited by the tube strength. The discrepancy of 20% between propulsive pressure and the structural pressure represents a loss of efficiency in some sense. If we now consider the charge-to-projectile mass ratio necessary to achieve a muzzle velocity of the order of 3 km/sec with a conventional granular charge we find that W is of the order of 8 so that $p_{base} \sim 0.2\ p_{breech}$. Evidently we have reached the regime of diminishing returns; much of the propulsive potential of the propellant is lost to self-work. A similar conclusion follows from an examination of the ratio of kinetic energy of the charge to that of the projectile, which takes the values 0.167 and 2.67 for the two cases we have considered.

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[2] *Krier, H. and Summerfield, M., "Interior Ballistics of Guns" Progress in Astronautics and Aeronautics, Vol. 66. American Institute of Aeronautics and Astronautics 1979*

[3] *Corner, J. "Theory of the Interior Ballistics of Guns" Wiley, New York, 1950*

Now let us consider the end-burning traveling charge configuration also illustrated in figure 1.1. The concept formulated by Langweiler[4] has the following novel aspect. The rate of burning of the propellant is to be controlled in such a fashion that the reacted gas comes to rest with respect to the frame of reference of the gun. Evidently, the burn rate is required initially to be zero and to increase as the projectile accelerates. At any instant, however, the kinetic energy of the gas is zero and, in particular, as burnout is approached, all the propellant is at rest, the kinetic energy being contained only in the projectile. Alternatively, as the gas column is always at rest, suffering neither translation nor dilation, its pressure remains constant at the initial value and is, moreover, uniform with respect to its length. The possibility that these observations can be translated into improved ballistic efficiency has motivated both past and present interest in the traveling charge.

Certain questions come to mind as regards this possibility. We first note those of previous investigators and then add some observations of our own.

The first question is whether the burn rates required by the Langweiler concept can actually be realized. It was observed by Vinti[5] that the regression rate would be required to be of the same order of magnitude as the projectile velocity, namely one or more km/sec. This requirement may be contrasted with the observed rates of burning of conventional propellants which are typically measured in cm/sec at the largest pressures experienced in a gun. The same observation was made by Lee and Laidler[6] who also commented on the extraordinary mechanical properties that would be required to transmit the propellant thrust to the base of the projectile. Apart from the technological difficulties of high strength and high burning rate, Lee and Laidler did not discern any fundamental obstacles to the implementation of the traveling charge concept.

Vinti noted that the pressure on the unreacted side of the gas/propellant interface would exceed that of the reactants and that the excess would be proportional, in some fashion, to the regression rate. The excess is, of course, the contribution of momentum flux to the total propulsive thrust. Accordingly, the Langweiler concept does not

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[4] *Langweiler, H. "A Proposal for Increasing the Performance of Weapons by the Correct Burning of Propellant" British Intelligence Objective Sub-Committee, Group 2, Ft. Halstead Exploiting Center, Report 1247* *undated*

[5] *Vinti, J. P. "Theory of the Rapid Burning of Propellants" Ballistic Research Laboratories Report No. 841* *1952*

[6] *Lee, L. and Laidler, K. J. "The Interior Ballistics of the Impulse Propulsion Gun" Nord 10260 Contract Report CU/F/52.2,* *August 1951*

operate on a constant pressure cycle. The pressure at the base of the propellant increases with travel and it is this pressure, moreover, which the tube must support. In Vinti's opinion, the resulting combination of maximum pressure and muzzle velocity would not differ appreciably from that obtained using a conventional charge.

A more serious objection to the traveling charge concept of Langweiler was also expressed by Vinti. He observed that as the projectile velocity increased to a value exceeding the speed of sound in the reactants, the law of burning would be required to yield supersonic reactants, as perceived by an observer moving with the regressing surface. From the theory of the steady deflagration of a gas it has been shown, for all models of flame structure which have been investigated[7,8,9,10], that such a process cannot occur. It was concluded by Vinti, therefore, that the Langweiler concept would be inherently incapable of realization once the velocity of the projectile exceeded approximately 1.0-1.5 km/sec, which is the velocity of sound in present day propellant formulations.

We now add to these earlier caveats some considerations of our own. First we observe that the traveling charge is represented as self-supporting. It is not difficult to show that alternative designs in which the propellant is supported by some structural member lead to rather unattractive payload capacities, most of the projectile weight being concentrated in the propellant support system. If the propellant is unsupported, and required to transmit compressive stresses of the order of 500 MPa, we may expect its mechanical response to enter the hydrodynamic regime. That is to say, the components of the stress deviator are not expected to exceed the yield strength--typically of the order of 5-10 MPa in current compositions--so that the stress tensor can be regarded as isotropic, to a good degree of approximation. This implies that the radial stress exerted on the gun tube by the propellant will be approximately equal to the axial stress--as already assumed by Vinti. Therefore the problem of mechanical resistance due to wall friction must surely be considered.

Consider next the conditions as the projectile approaches the muzzle of the gun. Suppose that the ideal combustion model of Langweiler has indeed been realized. The pressure of the gas is therefore some large value. Yet the design constraints associated with muzzle blast require that the pressure be suitably limited. Therefore, either the initial gas pressure must be confined to that limit or burnout must occur prior to muzzle exit.

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[7] *Friedrichs, K. O. "On the Mathematical Theory of Deflagrations and Detonations" NAVORD Report 79-46 1946*

[8] *Courant, R. and Friedrichs, K. O. "Supersonic Flow and Shock Waves" Interscience, New York 1948*

[9] *Williams, F. A. "Combustion Theory" Addison-Wesley 1965*

[10] *Landau, L. D. and Lifschitz, E. M. "Fluid Mechanics" Pergamon Press 1959*

If burnout occurs, there will be imposed on the gas column a rarefaction wave which will accelerate it towards the muzzle. If enough projectile travel occurs between burnout and muzzle exit, the velocity profile will resume the distribution characteristic of the conventional charge. From the point of view of a global energy balance, therefore, the potential ballistic benefit of the traveling charge is apparently diminished following burnout of the propellant.

As a final observation we note that when burnout occurs, the velocity of the gas is required instantaneously to adjust to that of the projectile. Since the jump in velocity may be of the order of the speed of sound, the concomitant drop in pressure may be very large. Two consequences follow from this observation. First, the propulsive capacity of the gas after burnout will be diminished due to the sharp drop in base pressure. Second, a strong tensile wave will be transmitted through the projectile and mechanical failure might be a concern if sensitive fuzing or guidance components are present.

The loss of propulsive capacity following burnout suggests that the optimum use of a traveling charge will involve a design in which burnout occurs close to the muzzle. If the pressure drop associated with burnout is sufficiently great, it may be possible to satisfy the constraint associated with blast while operating from a high initial pressure.

From the foregoing discussion it is apparent that the evaluation of the potential merits of the traveling charge concept must be performed in the context of a model capable of recognizing the hydrodynamic implications of certain hypothetical or ideal combustion laws. Specifically, we seek herein to provide a model which is capable of answering questions of the following type. What are the implications, in regard to performance, of limiting the Langweiler combustion model to subsonic or sonic conditions? As the Langweiler combustion model does not in fact yield a constant base pressure on the projectile, would a more favorable ideal combustion model be one in which the stress in the unreacted propellant, or the acceleration of the projectile, were held constant? Would such an ideal law require supersonic combustion? If so, what would be the losses associated with a limitation to subsonic or sonic states in such cases?

1.2 Objectives and Scope of Present Study

The objectives of the present study are to formulate, encode and demonstrate a model of an end-burning traveling charge. The model is required to be of sufficient scope as to permit an assessment of the hydrodynamic and ideal combustion limits on performance. However, parametric studies to evaluate the theoretical performance of an end-burning traveling charge and the formation of comparisons with conventional charge performance are not within the scope of the present effort.

1.3 Summary of Approach

We first summarize the physical aspects of the model. Subsequently, we will comment on the method of solution. The model is predicated on the nature of the flow illustrated schematically in figure 1.1. The flow is assumed to be one-dimensional without change of area. The gas is assumed to obey a covolume equation of state and is modeled as a one-dimensional, inviscid, unsteady flow with an allowance for heat loss to the tube. The propellant may be represented either as rigid or as a viscoplastic continuum in which an allowance is made for the friction exerted on the tube.

The breech boundary condition may be represented either as closed or as permeable to the gas. The latter case enables the application of the model to vented chambers and to recoilless rifles. The code also permits a simulation of the blowdown of the tube following the expulsion of the projectile and any unburned propellant.

The interface between the gas and the solid propellant is treated as a discontinuity. A variety of models are provided to describe the rate of burning. We have already noted that there are strong arguments to deny the possibility of a steady deflagration wave with supersonic reactants. These arguments are reviewed in Appendix A. We nevertheless provide models of both subsonic and supersonic deflagrations. Our purpose in providing the latter is principally to permit an assessment of the extent to which the theoretical performance of an end-burning traveling charge is limited by the restriction of burning to the subsonic or sonic conditions.

Considering first the subsonic cases it follows that only one condition may be imposed on the burning process. We admit any one of the following. The burn rate may be described as a function of pressure according to measurements. The burn rate may be required to yield the ideal combustion of Langweiler. The burn rate may be required to yield a predetermined value of pressure on the unreacted side of the gas/propellant interface or to yield a predetermined value of the acceleration of the projectile. Finally, the burn rate may be required to yield a predetermined Mach number of the reactants, provided, of course, that this value is less than one. In all these cases, the subsonic nature of the flow demands that the reactants satisfy a condition of mechanical compatibility with the column of gas.

If the reactants become supersonic, they are no longer required to be mechanically compatible with the gas. The deflagration influences but is not influenced by the motion of the gas unless a shock of sufficient strength is formed to overtake the supersonic boundary. We admit two possible models of a supersonic deflagration. The first is the Langweiler model. In the case of supersonic burning, two conditions may be imposed on the process. The Langweiler process may be interpreted as specifying both the velocity and pressure of the reactants. A second model is defined by specifying the Mach number of the reactants --greater than or equal to unity--and either the pressure on the unreacted side of the gas/propellant interface or the acceleration of the projectile.

Full details of the physical model and the relevant governing equations are given in chapter 2.0. In chapter 3.0 we describe the method of solution which may be summarized as follows. A time dependent mesh is allocated subject to constraints on minimum mesh size and total number of points. A two level explicit marching scheme is used to update the state of interior mesh points and the method of characteristics is used at the boundaries. Chapter 3.0 describes in detail the implementation of the various models of burning and also discusses the rules according to which branching may occur among them during a given interior ballistic cycle.

The method of solution is illustrated in chapter 4.0. A comparison is given with an exact solution for a simple case to provide an absolute benchmark of accuracy. A nominal traveling charge configuration is

discussed to exhibit the degree of mesh indifference of the code and the extent to which mass and energy are conserved on a global basis. We also comment briefly on the ballistic consequences of friction between the propellant and the tube.

The code itself is described in Appendix B which contains tables of the various routines and their linkages, a glossary of variable names, and the format of the data used to run the code. A Fortran IV listing is also attached.

2.0 GOVERNING EQUATIONS FOR MODEL

The governing equations for the model consist of one-dimensional time-dependent statements of the balances of mass, momentum and energy supported by constitutive laws and subjected to boundary and initial conditions. The gas is always treated as a continuum as described in section 2.1. The unreacted propellant may be represented either as a continuum or as a rigid body, according to the interests of the user of the model. The equations in the continuum case are described in section 2.2 while the case of rigid body motion is included with the discussion of the motion of the projectile in section 2.3. The constitutive laws are described in section 2.4. The discussion of section 2.4 includes the equations of state for the gas and the solid propellant, heat loss to the tube, friction between the tube wall and the solid propellant, resistance to projectile motion and, finally, the description of non-ideal rates of burning such as those determined experimentally. The analysis of ideal burn rates, in which the reactants are required to come to rest or to attain a prespecified Mach number or to induce a predetermined level of acceleration of the projectile or of stress in the unreacted propellant, are described in section 2.5 which treats the boundary conditions. Finally, section 2.6 of this chapter discusses the initial conditions; when the propellant is burning vigorously at the initial instant, these may be non-trivial. The method of solution of the governing equations is the subject of chapter 3.0.

2.1 Balance Equations for Gas

The motion of the gas is assumed to be one-dimensional, unsteady, inviscid and non-heat-conducting. However, the loss of heat to the tube wall is considered in the balance of energy. Using t to represent time, x to represent the axial coordinate and ρ, u, p, e to represent the density, velocity, pressure and internal energy of the gas, the balance equations may be expressed in the usual forms[8] :

Balance of Mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \rho u = 0 \qquad 2.1.1$$

Balance of Momentum

$$\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x} (\rho u^2 + g_o p) = 0 \qquad 2.1.2$$

Balance of Energy

$$\frac{\partial}{\partial t} \rho\left(e + \frac{u^2}{2g_o}\right) + \frac{\partial}{\partial x} \rho u\left(e + \frac{p}{\rho} + \frac{u^2}{2g_o}\right) = -q_w \qquad 2.1.3$$

Here we have also used q_w to represent the heat loss per unit volume of gas per unit time and g_o is a constant used to reconcile units.

It is well known that these equations are of the hyperbolic type[8]. That is to say, there exist real lines on which the equations reduce to ordinary differential forms involving only derivatives along the lines in question. The lines are referred to as characteristics and the differential forms are called conditions of compatibility. The existence of the characteristic lines follows naturally from a consideration of the Cauchy problem for the quasi-linear system[11]

$$A \frac{\partial \psi}{\partial t} + B \frac{\partial \psi}{\partial x} = C \qquad 2.1.4$$

where ψ and C are k-dimensional vectors and A and B are k x k matrices. Consider the transformation $(x,t) \rightarrow (s,n)$ so that 2.1.4 becomes

$$[An_t + Bn_x]\psi_n = C - [As_t + Bs_x]\psi_s \qquad 2.1.5$$

where the subscript denotes a partial derivative. Let the vector ψ be specified on the initial line $n = n_o$. Then the line is called free if 2.1.5 permits the determination of the normal derivatives and characteristic if it does not. The importance of this point is simply that if the normal derivatives can be determined, successive differentiation will permit the determination of derivatives of all orders with respect to n so that the solution can be obtained in a neighborhood of the initial data by means of Taylor's theorem.

Evidently, the condition that the line $n = n_o$ be free is that the rank of the matrix

$$\Delta = An_t + Bn_x$$

be $r(\Delta) = k$ whereupon 2.1.5 has a unique solution for the components of ψ_n. When $r(\Delta) < k$ the line $n = n_o$ is characteristic. However, if the equation 2.1.5 is to hold at each point on such a line, the initial data are not arbitrary, but are constrained by the condition of solvability for the linear system 2.1.5, namely

$$r(\Delta^+) = r(\Delta)$$

where $\Delta^+ = [\Delta;\ C - [As_t + Bs_x]\psi_s]$ is the augmented matrix. These conditions of solvability yield the conditions of compatibility for each characteristic line.

A convenient choice for the parameter s is $s = t$. However, it must be borne in mind that ψ_s is a derivative with n held constant so that $\psi_s = \psi_t t_s + \psi_x x_s$. One may also use the relations $dx/dt = x_s/t_s = -n_t/n_x$, where dx and dt are understood to be differentials along the line $n = n_o$, to recast Δ and Δ^+ as:

$$\Delta = Adx - Bdt \qquad 2.1.6$$

$$\Delta^+ = [\Delta;\ C - A \frac{d\psi}{dt}] \qquad 2.1.7$$

---

[11] *Petrovsky, I. G. "Partial Differential Equations" Interscience, New York 1954*

By eliminating e in favor of p and ρ in 2.1.3 and introducing the isentropic sound speed c, it is easy to to show that the system 2.1.1, 2.1.2, 2.1.3 yields the following characteristics and conditions of compatibility. On the lines designated by $\Gamma_g^{\pm}$ whose slopes satisfy

$$\frac{dx}{dt} = u \pm c \qquad 2.1.8$$

we have the conditions of compatibility

$$\frac{dp}{dt} \pm \frac{\rho c}{g_o}\frac{du}{dt} = -\frac{q_w}{\rho\left(\frac{\partial e}{\partial p}\right)_\rho} \qquad 2.1.9$$

while on the line Γ_g^{o} whose slope satisfies

$$\frac{dx}{dt} = u \qquad 2.1.10$$

we have the condition of compatibility

$$\frac{dp}{dt} - \frac{c^2}{g_o}\frac{d\rho}{dt} = -\frac{q_w}{\rho\left(\frac{\partial e}{\partial p}\right)_\rho} \qquad 2.1.11$$

We will refer to Γ_g^{+} and Γ_g^{-} as the acoustic characteristics and Γ_g^{o} will be referred to as the gas-material characteristic.

2.2 Balance Equations for Solid Propellant

When the propellant is represented as a continuum, it is taken to be one-dimensional, unsteady and isothermal. Accordingly, we require only the balances of mass and of momentum. We use ρ_p, u_p and σ to represent the density, velocity and pressure in the propellant. The balance of momentum also incorporates the influence of friction due to the contact of the propellant with the tube wall. We have

Balance of Mass

$$\frac{\partial \rho_p}{\partial t} + \frac{\partial}{\partial x}\rho_p u_p = 0 \qquad 2.2.1$$

Balance of Momentum

$$\frac{\partial}{\partial t}\rho_p u_p + \frac{\partial}{\partial x}\left[\rho_p u_p^2 + g_o\sigma\right] = f_w \qquad 2.2.2$$

The pressure is not assumed to be an integrable function of density since the stress response of the propellant is expected to be path dependent in general. We anticipate the discussion of section 2.4.2 by noting that we take the pressure and density to be related by

$$\frac{\partial \sigma}{\partial t} + u_p\frac{\partial \sigma}{\partial x} = \frac{a^2}{g_o}\left[\frac{\partial \rho_p}{\partial t} + u_p\frac{\partial \rho_p}{\partial x}\right] \qquad 2.2.3$$

where a is the rate of propagation of small disturbances and is, in general, dependent on both density and rate of change of density.

The procedure summarized in the previous section may be applied to equations 2.2.1, 2.2.2 and 2.2.3 to yield the following characteristic lines and conditions of compatibility. On $\Gamma_p^{\pm}$ defined by

$$\frac{dx}{dt} = u \pm a \qquad 2.2.4$$

we have the conditions

$$\frac{d\sigma}{dt} \pm \rho_p \frac{a}{g_o} \frac{du_p}{dt} = \pm a f_w \qquad 2.2.5$$

while on Γ_p^{o} defined by

$$\frac{dx}{dt} = u_p \qquad 2.2.6$$

we have

$$\frac{d\sigma}{dt} = \frac{a^2}{g_o} \frac{d\rho_p}{dt} \qquad 2.2.7$$

Equation 2.2.7 is, of course, just a restatement of equation 2.2.3.

2.3 Motion of the Projectile

The projectile is treated as a rigid body whose motion is resisted by the pressure of shocked air ahead of it and by friction between its obturating band and the tube wall. The resistive terms are described in section 2.4.5; here we simply represent the total resistive force by F. Using X_p to denote the position of the projectile at any time, M_p to denote its mass, A_b to denote the bore area of the tube and $\sigma(X_p)$ to denote the pressure at the interface between the solid propellant and the projectile base, the equation of motion of the projectile takes the simple form

$$M_p \ddot{X}_p = g_o [A_b \sigma(X_p) - F] \qquad 2.3.1$$

where a dot is used to denote a total derivative with respect to time.

In the preceding section we described the analysis of the propellant viewed as a continuum. If its mechanical response is not of interest it may be viewed as a rigid body and its motion incorporated with that of the projectile. Let the position of the regressing propellant surface be denoted by x_p, relative to the same origin as X_p; let the instantaneous mass of the propellant be m_p and denote by r the rate of regression relative to the unreacted propellant whose velocity is equal to that of the projectile, namely $\dot{X}_p$. Evidently

$$r = \dot{x}_p - \dot{X}_p \qquad 2.3.2$$

Moreover, letting ρ_{p_o} be the constant value of propellant density in the case when it is viewed as a rigid body, we clearly have

$$\dot{m}_p = - A_b \rho_{p_o} r \qquad 2.3.3$$

If $\sigma(x_p)$ is the pressure on the unreacted side of the regressing surface, it follows that the equation of motion of the propellant and the projectile, viewed as a single rigid body, is just

$$(M_p + m_p)\ddot{X}_p = g_o[A_b\sigma(x_p) - F] \qquad 2.3.4$$

Equation 2.3.4 can be deduced either from limiting arguments in which x_p is approached from the unreacted side or by considering a balance of momentum for a control volume which instantaneously envelopes the projectile and unreacted propellant. In the latter case one has

$$\frac{d}{dt}[\dot{X}_p(M_p + m_p)] = g_o[A_b\sigma(x_p) - F] - A_b\rho_p r\dot{X}_p$$

Then the substitution of 2.3.3 into this result yields 2.3.4.

2.4 Constitutive Laws

We use the term constitutive law in a broad sense here to denote not only the equations of state for the gas and the solid propellant but also the equations governing heat loss, friction and measured rates of burning.

2.4.1 Equation of State of Gas

The gas is assumed to obey a covolume of state. Thus we have

$$e = c_v T = \frac{p(1 - b\rho)}{(\gamma - 1)\rho} \qquad 2.4.1$$

where c_v is the specific heat at constant volume, b is the covolume, γ is the ratio of specific heats, and we have used T to denote the temperature. If R is the universal gas constant and M is the molecular weight of the gas we have $c_v = R/M/(\gamma - 1)$. Moreover, from 2.4.1, we have the partial derivative

$$\left(\frac{\partial e}{\partial p}\right)_\rho = \frac{1 - b\rho}{\rho(\gamma - 1)} \qquad 2.4.2$$

as required by equations 2.1.9 and 2.1.11. Finally, the isentropic sound speed follows as

$$c^2 = \frac{\gamma g_o p}{\rho(1 - b\rho)} \qquad 2.4.3$$

2.4.2 Equation of State of Solid Propellant

From our formulation of the equations of motion of the solid propellant viewed as a continuum, it is evident that we anticipate finite strains. As reflected by equation 2.2.3, we embed the equation of state into the functional dependence of the rate of propagation of small disturbances on density and the rate of change of density. The general constitutive model for the response of the propellant may be thought of as viscoplastic since 2.2.3 expresses a differential relationship between stress and strain. Moreover, the relationship need not be reversible. However, hereditary behavior is not

considered here. Viscoelastic or hereditary behavior requires that derivatives of different orders be applied to stress and strain in the constitutive relationship. For example, strain-rate may be proportional to stress so that elongation continues while the stress is held constant. The use of 2.2.3 implies that density, or strain, will change only under the action of a change in stress. Since propellants do exhibit viscoelastic properties at moderate rates of strain and at ambient temperature, the use of 2.2.3 implies that the characteristic time associated with the application of the load must be short in comparison with the internal relaxation times.

The equation for the rate of propagation of small disturbances has been assumed here to take a particularly simple functional form, namely

$$a = \begin{cases} a_1 \rho_p/\rho_{p_o} & \text{, loading} \\ a_2 & \text{, unloading or reloading} \end{cases} \qquad 2.4.4$$

Here ρ_{p_o} is the density at ambient conditions and a_1 is the corresponding rate of propagation of a compressive wave. If E is the modulus of compression of a laterally confined sample of propellant subjected to small loads, then $a_1 = \sqrt{g_o E/\rho_{p_o}}$. To be physically meaningful, the unloading or reloading wave speed a_2 should exceed the loading wave speed for all values of ρ_p. In fact 2.4.4 has been encoded so that if $a_2 = 0$, a reversible law is used based on the loading branch of 2.4.4.

Unloading, reloading and loading are defined by reference to the rate of compression $\dot{\rho}_p = \partial\rho_p/\partial t + u_p \partial\rho_p/\partial x$ and the nominal loading curve defined by

$$\sigma_*(\rho_p) = \frac{a_1^2 \rho_{p_o}}{3g_o} \left[\left(\frac{\rho_p}{\rho_{p_o}}\right)^3 - 1\right] \qquad 2.4.5$$

The propellant is unloading if $\dot{\rho}_p < 0$; loading if $\dot{\rho}_p > 0$ and $\sigma = \sigma_*$; reloading if $\dot{\rho}_p > 0$ and $\sigma < \sigma_*$.

Expressed in the form 2.4.5, the nominal loading curve is seen to be a special case of that proposed by Murnaghan[12] in connection with the finite deformation of solids; in the general case the value 3 is replaced by k in both the exponent and the pre-multiplying group of 2.4.5. The functional dependence of a on ρ_p and $\dot{\rho}_p$ has been encoded in a modular fashion so that modifications may be made easily when the availability of data so warrants.

2.4.3 Heat Loss to the Tube

We use a very simple model for the heat loss to the tube, it being understood that our interest in heat loss is confined to its ballistic consequences, particularly the degradation of muzzle velocity. The tube wall is treated as though its temperature remained constant at the initial value T_w. The heat loss is determined by means of an empirical correlation for the film or heat transfer coefficient based on fully

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[12] *Murnaghan, F. D. "Finite Deformation of an Elastic Solid." Dover, New York 1967*

developed turbulent flow in a pipe. Both the assumption of constant wall temperature and that relating to the heat transfer coefficient are very crude; however, they involve errors which tend to compensate one another. The wall temperature will, of course, increase substantially during the interior ballistic cycle. A consequence of this increase will be a reduction in heat transfer which is proportional to the difference between gas temperature and wall temperature. On the other hand, the boundary layer is expected to be very much thinner, due to the short time available for its development, than is represented by a correlation based on a fully developed flow, and the film coefficient correspondingly greater. Thus the assumption of constant wall temperature implies an overestimate of heat loss which is compensated by the underestimate based on a film coefficient for fully developed flow.

The film coefficient is represented in the empirical form[13]

$$h = \frac{\kappa}{D} [0.023 \, Re_D^{4/5} Pr^{2/5}] \qquad 2.4.6$$

where h is the film coefficient, κ is the thermal conductivity of the gas, D is the diameter of the tube, $Re_D = \rho |u| D/\mu$ is the Reynolds number based on D, $Pr = c_p \mu/\kappa$ is the Prandtl number of the gas and μ is the viscosity of the gas.

Then the heat loss per unit volume of gas is seen to be related to h according to

$$q_w = \frac{4}{D} (T - T_w) h \qquad 2.4.7$$

Using 2.4.6 and eliminating κ in favor of h and μ we have

$$q_w = Q_w \frac{c_p \mu Pr^{-3/5}}{D^2} Re_D^{4/5} (T - T_w) \qquad 2.4.8$$

where $Q_w = 0.092$ according to 2.4.6 and 2.4.7. The precise value of Q_w is, however, at the discretion of the user, to facilitate ballistic matching.

2.4.4 Friction Between Propellant and Tube

The solid propellant is not expected to be able to sustain the axial stresses induced by the base pressure without the support of radial confinement. Indeed, the strength of current formulations is such that the radial component of stress will be quite close in value to the axial stress at any point. Such hydrodynamic behavior is typical of solids stressed significantly beyond the material yield point. As a consequence, mechanical interaction between the solid propellant and the tube wall is expected to be important and, in particular, the resistance due to friction may be of importance in many cases.

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[13] *Holman, J. P. "Heat Transfer" McGraw-Hill, New York 1968*

In order to assess the ballistic losses associated with friction between the solid propellant and the tube we provide two simple models. The first model takes the frictional resistance f_w to be proportional to the local value of the pressure in the form

$$f_w = -\frac{4}{D}\mu_w(u_p)\sigma \ \mathrm{Sgn}(u_p) \qquad 2.4.9$$

where μ_w is a velocity dependent coefficient of friction and Sgn denotes the sign function.

Alternatively, if a gas film of viscosity μ_f and thickness δ_f can be interposed between the propellant and the tube wall, the resistance f_w becomes[10]

$$f_w = -\frac{4}{D}\frac{\mu_f}{\delta_f}u_p \qquad 2.4.10$$

and is proportional to velocity rather than pressure.

2.4.5 Resistance to Projectile Motion

The resistance to projectile motion is considered to stem from two sources. We write

$$F = A_b\,(p_a + p_{band}) \qquad 2.4.11$$

where p_a is the pressure exerted by the air in front of the projectile and p_{band} reflects the resistance due to the obturator or rotating band and is also expressed as a pressure. The resistance due to the air in front of the projectile is determined from the pressure behind a shock whose strength is such that the compressed gas has a velocity equal to that of the projectile at any instant. Then if the unshocked air is taken to be at rest with pressure p_o and speed of sound c_o, it follows that p_a is given by

$$p_a = p_o\left\{(1+\mu^2)\left[\frac{\dot{X}_p + [\dot{X}_p^{\,2} + 4(1-\mu^2)^2 c_o^{\,2}]^{1/2}}{2(1-\mu^2)c_o}\right]^2 - \mu^2\right\} \qquad 2.4.12$$

where $\mu^2 = (\gamma_a - 1)/(\gamma_a + 1)$ and γ_a is the ratio of specific heats of air.

The resistance due to the obturator may be given in one of two forms. Either it is prespecified as a tabular function of projectile travel or it follows from an estimate of the normal force between the obturator and the tube wall as follows.

Let σ_z and σ_r represent the axial and radial components of stress in the projectile and, following the usual convention, let them be positive in tension. Let the length of the obturator, ℓ_b be sufficiently small that we can typify adequately the state of stress by the values at its midpoint. Let M_b be the projectile mass which is supported by a section through the

midpoint of the obturator and let μ_{wb} be the coefficient of friction between the tube and the obturator. A balance of axial forces yields

$$\frac{\pi D^2}{4}\sigma_z - \frac{\pi D \ell_b}{2}\mu_{wb}\sigma_r + \frac{M_b}{g_o}\ddot{X}_p = 0 \qquad 2.4.13$$

On the other hand, taking the tube to be sufficiently stiff that radial strains do not occur, and supposing the behavior of the projectile to be elastic, the axial and radial strains are related according to[14]

$$\sigma_r = \frac{\nu}{1-\nu}\sigma_z \qquad 2.4.14$$

where ν is Poisson's ratio. Substitution of 2.4.14 into 2.4.13 yields the value of σ_r whereupon the resistive pressure due to the obturator takes the form

$$p_{band} = \frac{4\mu_{wb}\ell_b}{g_o \pi D}\,\frac{M_b \ddot{X}_p}{\frac{1-\nu}{\nu}\frac{D^2}{4} - \frac{\mu \ell_b D}{2}} \qquad 2.4.15$$

Equation 2.4.15 is appropriate to the case when there is no initial interference between the band and the tube. If such an interference does exist, we assume it to be characterized by an initial shot start pressure p_s. Then the value of p_{band} given by 2.4.15 is augmented by the quantity $\mu_{wb}(\dot{X}_p)p_s/\mu_{wb}(0)$ in which the coefficient of friction is assumed to depend on velocity.

2.4.6 Propellant Burn Rate (Measured)

It is assumed that if data are available to describe the burn rate of the propellant, the functional relationship has the usual form

$$r = B_1 + B_2 p^n \qquad 2.4.16$$

in which B_1, B_2 and n are parameters which may take different values for successive segments of the propellant. The value of p is assumed here to pertain to the reacted material. Burn rates of the form given by equation 2.4.16 are only well posed physically when the reactants have a subsonic velocity relative to the regressing surface. Further discussion of this point is contained in Appendix A.

2.5 Boundary Conditions

The boundary conditions are of several types and are discussed in several subsections. Conditions at the breech, which may be either closed or open, are discussed in section 2.5.1. In section 2.5.2 we consider first the conditions at the base of the projectile when the propellant has not burned out. Subsequently we consider the change in conditions

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[14] *Fung, Y. C. "Foundations of Solid Mechanics" Prentice-Hall, 1965*

at burnout. Finally, we note the conditions which apply following the exit of the projectile, if blowdown of the tube is of interest. The boundary conditions then apply to the open muzzle and the discussion is similar to that of section 2.5.1. The conditions at the regressing interface between the reacted and unreacted propellant are discussed in section 2.5.3. This section also addresses the analysis of the ideal rates of burning.

2.5.1 Breech (Closed or Open)

The boundary conditions at the breech are taken to apply only to the gas as the solid propellant is assumed never to approach the breech. When the breech is closed there is but one condition to consider, namely the vanishing of the velocity of the gas. If we take the breech as the origin of the axial coordinate x we therefore require

$$u(0,t) = 0 \quad , \text{ all } t \qquad 2.5.1$$

When the breech is open it is assumed to be connected to the exterior via a nozzle whose throat area is A_* and whose discharge coefficient is C_{DB}. We assume $C_{DB}A_* \leq A_b$. The external pressure is assumed to be negligible by comparison with the stagnation pressure at the boundary. Therefore, we do not consider the possibility of a totally subsonic efflux and the external venting area is of no concern.

The critical mass flow rate corresponding to sonic conditions at the throat may be approximated with sufficient accuracy by the equation[3]

$$\dot{m}_* = C_{DB}A_*p_{STAG}\sqrt{\frac{\gamma g_o M}{RT_{STAG}}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}\ \{1 - 0.224y + 0.104y^2\} \qquad 2.5.2$$

where $y = (bp_{STAG}M)/(RT_{STAG})$. This result is strictly true only for the case $\gamma = 1.25$, but we regard the evaluation of the covolume correction for other values of γ to be unnecessary in view of the smallness of the correction. The stagnation temperature and pressure are related to the boundary values according to

$$p_{STAG} = p\left(\frac{T_{STAG}}{T}\right)^{\frac{\gamma}{\gamma-1}} \qquad 2.5.3$$

and

$$T_{STAG} = T + \frac{u^2/2g_o + b(p - p_{STAG})}{c_p} \qquad 2.5.4$$

where the unsubscripted quantities represent the boundary values.

Because the flow is unsteady, it is not necessarily the case that the Mach number $M = |u|/c$ is limited to values less than or equal to unity. However, we suppose that quasi-steady arguments do apply to the gas between the boundary and the exterior. Then the rules governing the discharge may be stated as either $M > 1$ and $\dot{m} = \rho A_b |u| < \dot{m}_*$ or else $\dot{m} = \dot{m}_*$. As noted above, the case $M < 1$ and $\dot{m} < \dot{m}_*$ is not of interest since the external pressure is assumed too low to allow a totally subsonic discharge. Since discharge is expected to commence with a Mach number less than or equal to unity, a transition to supersonic venting requires that a sufficiently strong compression wave impinge upon the boundary, the strength increasing with the excess of A_b over $C_{DB}A_*$. In principle, if $A_b > C_{DB}A_*$, a shock is required.

A different situation arises in the case of venting through the muzzle after the projectile has exited. Then the flow may initially be supersonic as we comment further in the next section.

2.5.2 Projectile Base and Muzzle Following Projectile Exit

Considering the conditions which apply at the base of the projectile it is clear that prior to burnout, when the propellant is represented as a continuum, we have

$$u_p(X_p,t) = \dot{X}_p \qquad 2.5.5$$

and, following burnout of the propellant, we have the corresponding condition on the gas, namely

$$u(X_p,t) = \dot{X}_p \qquad 2.5.6$$

The transition from 2.5.5 to 2.5.6 occurs at the instant of burnout. Because the sudden application of 2.5.6 to the gas may represent an instantaneous increase in velocity comparable in magnitude to the speed of sound, it is accompanied by a rather large drop in pressure. This point is discussed further in chapter 3.0 when we consider the numerical determination of the boundary values.

In some instances it will be of interest to continue the solution following the exit of the projectile from the muzzle. The histories of muzzle pressure and temperature are of interest since they are connected with problems of blast and flash. In almost all cases of interest the flow will be supersonic at exit, provided that burnout has occurred. The discharge at the muzzle is presumed to be characterized by a discharge coefficient C_{DMUZ}. Critical flow is determined by equation 2.5.2 with C_{DMUZ} in place of C_{DB} and A_b in place of A_*. If $C_{DMUZ} = 1$, and the discharge is initially supersonic, it will remain so until the Mach number reaches the value unity whereupon 2.5.2 will govern the discharge and the Mach number will remain equal to one. As with the treatment of the breech, subsonic venting is not considered. If the discharge is initially supersonic but $C_{DMUZ} < 1$, an abrupt transition will occur when the Mach number

decreases to a point at which the flow cannot be passed through the effective throat area $A_b C_{DMUZ}$. Physically, the occurrence of choking will result in the rearward propagation of a shock or strong compression wave to decelerate the supersonic flow. The magnitude of the jump will be proportional to the degree of departure of C_{DMUZ} from unity.

On the other hand, if the discharge is initally subsonic, choking will occur at the instant of expulsion of the projectile and a rarefaction will be propagated rearward to accelerate the flow.

2.5.3 Gas/Propellant Interface

The layer in which the unreacted propellant is thermally stimulated, decomposes, and releases its energy of chemical bonding, is assumed to be sufficiently thin that it may be represented as a surface of discontinuity. Further discussion of this assumption is given in Appendix A. Using the previously established nomenclature, we may express the principles of conservation of mass, momentum and energy for the propellant transported across the surface of discontinuity in the following forms.[8]

$$\rho(u - \dot{x}_p) = - \rho_p r \qquad 2.5.7$$

$$p + \frac{\rho}{g_o} (u - \dot{x}_p)^2 = \sigma + \frac{\rho_p}{g_o} r^2 \qquad 2.5.8$$

$$e + \frac{p}{\rho} + \frac{1}{2g_o} (u - \dot{x}_p)^2 = e_p + \frac{\sigma}{\rho_p} + \frac{r^2}{2g_o} \qquad 2.5.9$$

Here e_p may be understood to mean the chemical energy released following decomposition of the propellant. We also note

$$\dot{x}_p = u_p + r \qquad 2.5.10$$

which is identical with 2.3.2 when the propellant is taken to be rigid. Of course, if the propellant is not burning as may be required in certain ideal cases, 2.5.7, 2.5.8 and 2.5.9 are replaced by the conditions of continuity of pressure and velocity.

Because the regressing surface is presumed to be a deflagration wave, its velocity of advance relative to the unreacted propellant must always be subsonic[8]. Accordingly, when the propellant is treated as a continuum, it follows that the characteristic lines Γ_p^o and Γ_p^- both intercept the regressing surface and impose conditions of compatibility between the boundary values on the unreacted side and values in the interior of the propellant. Further discussion of the application of this observation is given in chapter 3.0. Because of the two conditions of compatibility, the three quantities σ, u_p and ρ_p are not all independent of the state of the interior. We may regard σ as the independent member which, when specified, yields unique values of u_p and ρ_p which are compatible with the interior state of the propellant. On the other hand, if the propellant is treated as rigid we have $\rho_p = \rho_{p_o}$, a constant, and $u_p = \dot{x}_p$ which is, in turn, a function of σ. Accordingly, we may consider σ as the only unknown quantity pertaining to the state of the unreacted

propellant and that u_p and ρ_p are either given or follow from the conditions of compatibility. From this perspective therefore, we have five independent quantities to be determined at the interface, namely p, ρ, u, σ and r. The internal energy e is, of course, given as a function of p and ρ and $\dot{x}_p$ is related to u_p and r via equation 2.5.10. Thus we have three conditions, equations 2.5.7, 2.5.8 and 2.5.9 to determine five quantities. Clearly, we need two additional independent relations.

When the reactants are subsonic, as viewed from the regressing surface, it follows that characteristics of the Γ_g^+ family intercept the boundary and impose a condition of compatibility with the state of the interior of the gas. Thus, whenever the reactants are subsonic there remains just one additional condition which may be applied to the boundary values independently of the principles of conservation of mass, momentum and energy and of the requirement that the state of each side of the interface be compatible with the contiguous substance.

In the event that the reactants are subsonic we consider that the remaining boundary condition may be any one of the following.

(a) Measured Burn Rate (With Subsonic Reactants)

When measurements of the burn rate as a function of the pressure of the fully reacted gas are available, the burning rate law, in the form 2.4.16 completes the determination of the boundary values.

(b) Ideal Burning of Langweiler (With Subsonic Reactants)

According to the ideal traveling charge model of Langweiler[4] the reactants are to be brought to rest once reaction is complete. The condition u = 0 serves to complete the determination of the boundary values.

(c) Prespecified Value of Pressure on Unreacted Side or of Acceleration of Projectile (With Subsonic Reactants)

It is easy to see that if the ideal combustion of Langweiler commences from some initially quiescent state then the pressure of the reactants remains constant in time and uniform over the length of the gas column. However, the pressure on the unreacted side of the interface exceeds that in the gas by an amount which increases with the velocity of the projectile, the excess being due to the momentum jump at the interface and representing the contribution of momentum flux to the total thrust. From this point of view, therefore, the Langweiler concept departs from the traditional concept of an ideal interior ballistic cycle. The continual increase in the pressure of the unreacted propellant implies that the tube must be able to support a value which is higher than the average value used to propel the projectile. In addition, while the pressure is increasing, the total mass of the propelled

body, namely the projectile plus the unreacted propellant, is diminishing due to consumption of the propellant. Thus the acceleration of the projectile increases even faster than the pressure. In some cases the maximum allowable acceleration of the projectile may be constrained by structural considerations.

A direct statement of the traditional concept of a constant base pressure gun may be expressed by requiring that the pressure on the unreacted side of the interface be equal to some predetermined value. Such a condition then allows the determination of all the boundary values. If, alternatively, the structural limitations of the projectile are of concern, it may be desired to hold the acceleration equal to some limiting value. When the propellant is treated as a rigid body, equation 2.3.1 enables the direct translation of the condition on acceleration into a condition on the instantaneous value of the pressure on the unreacted side of the interface whereupon all the boundary values may be determined.

When the propellant is treated as a continuum it does not necessarily follow that the maximum pressure will occur at the interface since transient phenomena govern the distribution throughout the unreacted propellant. The pressure on the unreacted side may nevertheless be prespecified and, once given, will allow the determination of all the boundary values.

Similarly, the use of 2.3.1, in the case when the propellant is treated as a continuum, enables an instantaneous value of the pressure on the unreacted side to be determined. However, it no longer follows that the acceleration of the projectile will be equal to the desired value.

Therefore, while we have encoded the possibility of an ideal combustion which yields either a predetermined value of pressure on the unreacted side or an equivalent value based on equation 2.3.1 and a prespecified value of acceleration, it should be understood that only when the propellant is treated as rigid will the desired ballistic consequence--control of maximum pressure or of projectile acceleration--be attained precisely. Transient phenomena, which will be determined by a continuum representation, may defeat the intended objective of the ideal law.

(d) Predetermined Mach Number of Reactants (Subsonic)

A final condition which we have considered is that the Mach number of the reactants be equal to some predetermined value. Given such a value, less than unity, all the boundary values may be found.

The preceding discussion has addressed several different conditions which may be applied, singly, to complete the specification of the boundary values when the reactants are subsonic. It may be of interest, as already mentioned in chapter 1.0, to consider an interior ballistic cycle in which the combustion is limited by more than one of the foregoing, the most limiting being considered at each stage of the cycle, with branching from one to another as the cycle unfolds. Further discussion of the manner in which the branching is conducted is given in chapter 3.0.

To conclude the present discussion we comment on the conditions to be considered when the reactants are supersonic as viewed from the regressing interface. As discussed in Appendix A, there are strong arguments to suggest that a steady combustion process with supersonic reactants cannot occur or, if it does occur, is completely unstable. Possibly, if the flame is sufficiently thick and sufficiently unsteady, supersonic reactants could result. In any case, it may be of interest to ascertain whether there is indeed any ballistic benefit to be gained from a combustion process with supersonic reactants. For this purpose we have encoded conditions which do admit the possibility of a so-called strong deflagration wave.

When the reactants are supersonic it is no longer the case that they have to be compatible with the flow in the interior of the gas. They influence, but are not influenced by the contiguous fluid as signals do not propagate from the interior to the boundary once the flow at the boundary becomes supersonic. In contrast to the preceding discussion of the subsonic case, therefore, we require two independent conditions to determine all the boundary values. We consider the two following combinations.

(a) Ideal Burning of Langweiler (With Supersonic Reactants)

When the reactants are subsonic and the initial state is quiescent and uniform, the ideal combustion of Langweiler involves the single requirement $u = 0$ whereupon the condition of compatibility on Γ_g^+ yields $p = p_{ST}$ where p_{ST} is the initial pressure of the quiescient gas column. When the reactants are supersonic, members of Γ_g^+ no longer intercept the regressing surface and the condition $u = 0$ does not imply $p = p_{ST}$ necessarily. However we may require $p = p_{ST}$ as an independent condition on the supersonic reactants. Thus in the supersonic case, the ideal burning of Langweiler is described by two conditions $u = 0$ and $p = p_{ST}$ whereupon all the boundary values may be determined and, moreover, the gas column remains quiescent.

(b) Prespecified Value of Pressure on Unreacted Side or of Acceleration of Projectile (With Supersonic Reactants)

The previous discussion of this case subject to subsonic reactants remains unchanged insofar as the

relationship between the pressure and the acceleration limitations is concerned. Therefore we predicate our subsequent discussion on the assumption that the pressure on the unreacted side is predetermined. To complete the determination of the boundary values we require an additional relationship which replaces the condition of compatibility on Γ_g^+. We assume that this condition is furnished as a predetermined value of the Mach number $M \geq 1$. It should be noted therefore that this case does not require the regularity of the gas phase properties which results from the continuation of the Langweiler combustion into the supersonic regime.

2.6 Initial Conditions

The initial values for these quantities governed by ordinary differential equations are the intuitively natural set. The projectile is initially at rest, its position is known, and the propellant has some predetermined initial mass. In cases when parametric studies are to be performed, it may be desirable to compute these initial values so as to satisfy certain system constraints. In many cases of interest it is desirable to assume that the ignition charge is of sufficient energy as to elevate the chamber pressure to several hundred MPa, the initial pressure being equal to the maximum pressure. In such cases, the contribution of the igniter to the total mass and energy of the propelling charge may be significant and must be accounted for in a parametric study based, say, on a constant ratio of charge mass to projectile mass. We therefore note some relationships among the charge parameters which are useful in describing the initial conditions according to various alternative schemes. The schemes admitted by the code are described in Appendix B.

Let the initial volume of the gas be $V = A_b x_{b_o}$ where x_{b_o} is the length of the column at the initial instant. Let p_{ST} be the initial pressure. Let m_{g_o} and m_{p_o} be the initial masses of the gas and the solid propellant and let W be the charge to projectile mass ratio. If we assume that the ignition gas is of the same composition as the propellant and that it has been fully decomposed without the performance of external work or heat loss, its internal energy will be the energy of bonding e_p. Thus we have

$$m_{g_o} = V\rho = \frac{V}{(\gamma - 1)\frac{e_p}{p_{ST}} + b} \qquad 2.6.1$$

Therefore the following relationship holds between the mass of the projectile and the total mass of the propelling charge

$$WM_p = m_{p_o} + \frac{V}{(\gamma - 1)\frac{e_p}{p_{ST}} + b} \qquad 2.6.2$$

When an ideal burn rate is used such that the regression rate is zero at the initial instant, the initial acceleration a is given by

$$a = \frac{A_b g_o p_{ST}}{M_p + m_{p_o}} \qquad 2.6.3$$

provided that the propellant is viewed as rigid. Substitution of 2.6.3 into 2.6.2 yields a formula which relates the initial pressure to given values of a and W, namely

$$p_{ST} = -\frac{1}{2b}\left[K_1 - \sqrt{K_1^2 + K_2}\right] \qquad 2.6.4$$

where

$$K_1 = \frac{Va}{A_b g_o} + (\gamma - 1)e_p - \frac{ab}{A_b g_o}(1 + W)M_p$$

$$K_2 = 4\frac{(\gamma - 1)ab}{A_b g_o} e_p M(1 + W)$$

It should be kept in mind that 2.6.1 and 2.6.4 which depend on the assumption that the initial energy of the gas is equal to e_p are only strictly true when the regression rate is initially zero, as will be clear from the subsequent discussion of the continuum variables.

In principle, the initial conditions for the continuum variables should express the state of the combustion chamber and the solid propellant directly after loading into the gun. That is to say, the gas should be air at ambient temperature and pressure, the propellant should be at ambient pressure, and both should be at rest. The model should then reflect the influence of the igniter products which simultaneously pressurize the chamber and provide a thermal stimulus to the propellant. However, the present model does not reflect explicitly the influence of the igniter. The characteristics of the igniter are taken to be embedded directly into the initial conditions.

In those cases in which the propellant is taken to have a non-zero burning rate at the initial instant, either due to a prespecified pressure dependent burning rate or as a consequence of satisfying the predetermined condition on the pressure on the unreacted side of the interface, the initial velocity distribution in the gas is taken to vary linearly from the value zero in the breech to the value determined on the fully reacted side of the interface. This provision eliminates the necessity for dealing with a non-analytic initial condition. The pressure and density are, however, uniform and are taken to be equal to the boundary values on the reacted side of the interface.

The solid propellant is always taken to be at rest. However, when wall friction is considered we assume that the initial distribution of pressure is such as to satisfy the condition of mechanical equilibrium.

Then if σ is the value at the unreacted side of the interface, the pressure within the propellant is given by the distribution

$$\sigma(x) = \sigma \exp\left\{-\frac{4\mu}{D}(x - x_p)\right\} \qquad 2.6.5$$

where μ is the coefficient of friction corresponding to $u_p = 0$. This initial distribution reflects the possibility of lockup of the charge when its aspect ratio is sufficiently large.

The initial length of the solid propellant depends on the distribution of pressure. Evidently we have the relation

$$m_{p_o} = A_b \int_{x_p}^{X_p} \rho_p(x)\,dx \qquad 2.6.6$$

where ρ_p is assumed to be related to σ according to the nominal loading curve 2.4.5. The length is determined iteratively using a midpoint search so that more complicated constitutive data can be accommodated without changing the method.

3.0 METHOD OF SOLUTION

Having summarized the governing equations for the model, we turn now to the method of solution. The method of solution may be summarized as follows. The physical domain occupied instantaneously by the gas is mapped onto a unit line to yield a stationary equally spaced mesh for the purpose of obtaining a finite difference solution of the balance equations. The same is true of the domain occupied by the solid propellant when it is represented as a continuum. In each case the solution is advanced in time by means of a two level predictor/corrector algorithm which uses the physical balance equations at the interior mesh points and the characteristic forms at the boundaries.

The mesh allocation algorithm is described in section 3.1 and the computational form of the equations is described in 3.2. The procedures for interior and boundary mesh points are described in sections 3.3 and 3.4 respectively. The chapter concludes with a discussion of special considerations, such as the treatment at burnout, in section 3.5.

3.1 Mesh Allocation

The mesh is allocated dynamically, the number of mesh points varying from time to time in accordance with the following algorithm. The user specifies two parameters which we refer to by their Fortran names, see Appendix B, namely DXMIN, the minimum allowable mesh spacing in the physical plane, and MAXDIM, the maximum allowable total number of mesh points to be used in the calculation. Then a total of NDIM mesh points is allocated to the representation of the gas and a total of NDIM2 -NDIM mesh points is allocated for the representation of the propellant, if it is treated as a continuum. The allocation is subject to the following constraints.

$$\frac{x_p}{NDIM - 1} \geq DXMIN \qquad 3.1.1$$

$$\frac{X_p - x_p}{NDIM2 - NDIM1} \geq DXMIN \qquad 3.1.2$$

where NDIM1 = NDIM + 1, and the total number NDIM2 is required to satisfy

$$NDIM2 \leq MAXDIM \qquad 3.1.3$$

If the propellant is not treated as a continuum, 3.1.2 is not used and 3.1.3 is replaced by NDIM ≤ MAXDIM. In addition, it is required that each continuum region be sufficiently large that 3.1.1 and 3.1.2 will admit at least three mesh points.

When the propellant becomes so short, as a consequence of burning, that 3.1.2 will no longer admit three or more mesh points for its representation, the code automatically resets an internal switch and the calculation is concluded with the propellant treated according to a lumped parameter formulation. Further discussion of this point is given in section 3.5.

The constraints 3.1.1 and 3.1.2 are used first to establish a total number of mesh points to yield a resolution DXMIN. If 3.1.3 is satisfied, the values NDIM, NDIM2 given by 3.1.1 and 3.1.2 are used to represent the solution. If 3.1.3 is violated, the total number MAXDIM is allocated on a pro rata basis in accordance with the numbers proposed by 3.1.1 and 3.1.2.

The mesh allocation algorithm is exercised at the beginning of each integration step. From time to time the total number of mesh points allocated to a region may vary. It then becomes necessary to interpolate the existing arrays in order to create the new representation of the solution.

We use the cubic spline interpolation scheme of Walsh et al[15], simplified to take advantage of the fact that the data are equally spaced. Let $\phi(x)$ be the interpolating function for data $y_j = y(x_j)$, $j = 1,\ldots,n$ and $x_{j+1} - x_j = d$ where d is a constant. Let $\mu_j = \phi''(x_j)$, then we have the interpolation formula

$$\phi(x) = \{(x - x_j)y_{j+1} + (x_{j+1} - x)y_j\}/d \qquad 3.1.4$$

$$- (x - x_j)(x_{j+1} - x)\{(d + x_{j+1} - x)\mu_j + (d + x - x_j)\mu_{j+1}\}/6d$$

One may show that if we take $S_1 = 1$ and put

$$S_j = 1 - 1/(16S_{j-1}) \qquad 3.1.5$$

$$Z_j = \frac{3}{2d^2}(y_{j+1} + y_{j-1} - 2y_j) \qquad 3.1.6$$

then, defining $Z_2' = Z_2$ and putting

$$Z_j' = Z_j - \frac{Z_{j-1}'}{4S_{j-2}}, \quad j > 2 \qquad 3.1.7$$

the values of μ_j follow as $\mu_1 = \mu_n = 0$ and

$$\mu_{n-1} = Z_{n-1}'/S_{n-2} \qquad 3.1.8$$

$$\vdots$$

$$\mu_j = (Z_j' - \tfrac{1}{4}\mu_{j+1})S_{j-1} \qquad 3.1.9$$

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[15] *Walsh, J. L., Ahlberg, J. H. and Nilson, E. N., "Best Approximation Properties of the Spline Fit"*
J. Math. Mech. 11, 225-234 1962

3.2 Transformed Equations

Taking the origin to be at the breech it follows that the gas may be mapped onto a unit line by the transformation

$$\zeta = x/x_p \tag{3.2.1}$$

It follows that the balance equations must be transformed according to the rules

$$\frac{\partial}{\partial x} \to \frac{1}{x_p} \frac{\partial}{\partial \zeta} \tag{3.2.2}$$

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} - \frac{\zeta \dot{x}_p}{x_p} \frac{\partial}{\partial \zeta} \tag{3.2.3}$$

We also note that for arbitrary ψ

$$\frac{1}{x_p} \frac{\partial}{\partial \zeta} [\psi(u - \eta)] = \frac{1}{x_p} \frac{\partial}{\partial \zeta} \psi u - \frac{\eta}{x_p} \frac{\partial \psi}{\partial \zeta} - \psi \frac{\dot{x}_p}{x_p} \tag{3.2.4}$$

and where we have introduced $\eta = \zeta \dot{x}_p$.

We now introduce the dependent variables

$$G_1 = \rho x_p \tag{3.2.5}$$

$$G_2 = G_1 u = \rho u x_p \tag{3.2.6}$$

$$G_3 = G_1(e + u^2/2g_o) = \rho x_p(e + u^2/2g_o) \tag{3.2.7}$$

The transformed, or computational, form of the balance equations 2.1.1, 2.1.2 and 2.1.3 is then seen to be as follows.

$$\frac{\partial G_1}{\partial t} + \frac{\partial}{\partial \zeta} \left[\frac{G_1}{x_p} (u - \eta) \right] = 0 \tag{3.2.8}$$

$$\frac{\partial G_2}{\partial t} + \frac{\partial}{\partial \zeta} \left[\frac{G_2}{x_p} (u - \eta) + g_o p \right] = 0 \tag{3.2.9}$$

$$\frac{\partial G_3}{\partial t} + \frac{\partial}{\partial \zeta} \left[\frac{G_3}{x_p} (u - \eta) + pu \right] = -q_w x_p \tag{3.2.10}$$

The balance equations for the solid propellant can be put into a form analogous to 3.2.8 and 3.2.9. However, because of the differential constitutive law 2.2.3, it is convenient to keep the total time derivative of ρ_p isolated. The computational form of the equations 2.2.1, 2.2.2 and 2.2.3 is as follows.

$$\frac{\partial \rho_p}{\partial t} + \frac{u_p - \eta_p}{X_p - x_p} \frac{\partial \rho_p}{\partial \zeta} + \frac{\rho_p}{X_p - x_p} \frac{\partial u_p}{\partial \zeta} = 0 \tag{3.2.11}$$

$$\frac{\partial u_p}{\partial t} + \frac{u_p - \eta_p}{X_p - x_p}\frac{\partial u_p}{\partial \zeta} + \frac{g_o}{\rho_p (X_p - x_p)}\frac{\partial \sigma}{\partial \zeta} = \frac{f_w}{\rho_p} \qquad 3.2.12$$

$$\frac{\partial \sigma}{\partial t} + \frac{u_p - \eta_p}{X_p - x_p}\frac{\partial \sigma}{\partial \zeta} = \frac{a^2}{g_o}\left\{\frac{\partial \rho_p}{\partial t} + \frac{u_p - \eta_p}{X_p - x_p}\frac{\partial \rho_p}{\partial \zeta}\right\} \qquad 3.2.13$$

where, by analogy with 3.2.1, we have $\zeta = (x - x_p)/(X_p - x_p)$ and $\eta_p = \dot{x}_p + \zeta(\dot{X}_p - \dot{x}_p)$.

3.3 Integration at Interior Mesh Points

The balance equations for the gas phase are integrated at interior mesh points using the two-level predictor/corrector scheme of MacCormack[16]. Let ψ_j^n be understood to mean the value of ψ at the j-th mesh point and the n-th step of the integration. Then 3.2.8, 3.2.9, 3.2.10 are put into a finite difference form according to the following rules of discretization.

$$\frac{\partial \psi}{\partial t} \rightarrow \begin{cases} (\tilde{\psi}_j - \psi_j^n)/\Delta t & \text{, predictor} \\ (\psi_j^{n+1} - 1/2(\tilde{\psi}_j + \psi_j^n))/(\Delta t/2) & \text{, corrector} \end{cases} \qquad 3.3.1$$

and

$$\frac{\partial \psi}{\partial \zeta} \rightarrow \begin{cases} (\psi_{j+1}^n - \psi_j^n)/\Delta \zeta & \text{, predictor} \\ (\tilde{\psi}_j - \tilde{\psi}_{j-1})/\Delta \zeta & \text{, corrector} \end{cases} \qquad 3.3.2$$

where $\Delta\zeta$ is the nondimensional mesh spacing in the computational plane and Δt is the time step through which the solution is being advanced. The substitution of 3.3.1 and 3.3.2 into 3.2.8, 3.2.9, 3.2.10 at the predictor level yields a system of linear algebraic equations for the predictor quantities $\tilde{\psi}_j$. Then the corrector level of the scheme is performed to yield the values ψ_j^{n+1}.

Somewhat different rules are used to perform the discretization of 3.2.11, 3.2.12 and 3.2.13. The rules for the time derivatives are as expressed by 3.3.1. However, first order upstream differencing is always used for the convective terms and the remaining spacewise derivatives are treated by centered differencing. That is

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[16] *MacCormack, R. W. "The Effect of Viscosity in Hypervelocity Impact Cratering" AIAA Paper 69-354 1969*

$$(u_p - \eta_p)\frac{\partial \psi}{\partial \zeta} = \begin{cases} (\psi_{j+1} - \psi_j)/\Delta\zeta & \text{if } u_p - \eta_p < 0 \\ (\psi_j - \psi_{j-1})/\Delta\zeta & \text{if } u_p - \eta_p \geq 0 \end{cases} \qquad 3.3.3$$

at both predictor and corrector levels. But

$$\frac{\partial \psi}{\partial \zeta} = \frac{\psi_{j+1} - \psi_{j-1}}{2\Delta\zeta} \qquad 3.3.4$$

applies to $\partial u_p/\partial\zeta$ in 3.2.11 and to $\partial\sigma/\partial\zeta$ in 3.2.12 at both predictor and corrector levels.

We also note that the rule expressed by 3.3.1 serves to define an algorithm for the integration of the ordinary differential equations such as 2.3.1, for example.

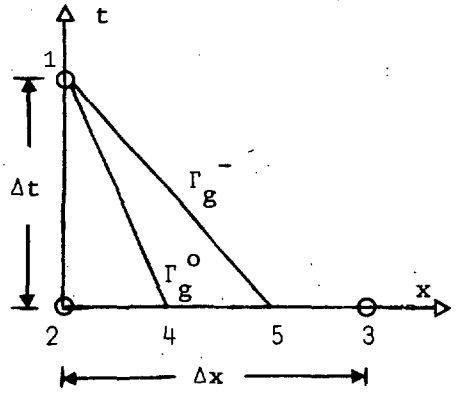
In order to assure stability of the method it is necessary that the time step be constrained. This point is discussed in section 3.5.

3.4 Integration at Boundary Mesh Points

The algorithms for the boundary points may be summarized in the following general form. The conditions of compatibility on those characteristic lines which intersect both the boundary and the line bearing the data at the present step are used to determine linear relationships between the boundary values of velocity and pressure and between the boundary values of density and pressure. These linear relationships are combined with the physical boundary conditions and the combination is then solved as a possibly nonlinear algebraic system, the method of solution varying in complexity in accordance with the nature of the boundary conditions.

We first indicate the manner in which the characteristic forms of the equations are used to establish linear relationships among certain of the boundary values, taking the gas-permeable breech as an example. The procedure at other boundaries is completely analogous. Subsequently, we will consider the impact of the physical boundary conditions, proceeding case by case through a number of subsections.

The sketch below illustrates the conditions pertinent to the subsonic efflux of gas through the breech and is expressed in terms of the physical coordinate x. The point 1 is understood to correspond to the state of the boundary at the next step of the integration. The points 2 and 3 correspond to the boundary and the adjacent mesh point at the present step. The point 5 corresponds to the intersection with the line of present data of the characteristic Γ_g^- which also intercepts the point 1. Similarly,



Γ_g^o, the material characteristic intercepts the boundary at 1 and the line of present data at 4. If the boundary were impermeable the point 4 would coincide with 2. If, on the other hand, the efflux were sufficiently intense as to become supersonic, there would also be a member of the family Γ_g^+ which would have an intersection with 1 and the segment 2-3.

Knowing Δx and Δt, the points of intersection 4 and 5 are deduced using 2.1.10 and 2.1.8 respectively, with the values of u and c determined at 2 on the predictor level and at 1 on the corrector level, the values in the latter case following from the predictor level solution. The conditions of compatibility 2.1.9 and 2.1.11 are interpreted as finite difference formulae with the state variables evaluated at 1 and 5 or at 1 and 4 as the case may be. The values of p, ρ, u at 4 and 5 are deduced from the values at 2 and 3 by linear interpolation. The coefficients of the differential terms in 2.1.9 are evaluated as averages of the values at 2 and 5 on the predictor step and 1 and 5 on the corrector step. A similar procedure is followed in the case of equation 2.1.11. No iteration is involved.

From this analysis it is easy to see that 2.1.9 will yield a linear relationship of the form

$$p = \alpha u + \beta \qquad 3.4.1$$

Similarly, 2.1.11 will yield a relationship of the form

$$\rho = \theta p + \phi \qquad 3.4.2$$

where p, u, ρ are all understood to be values at 1. Equation 3.4.2 remains valid if the boundary becomes impermeable but may not be used if the flow corresponds to influx. Similarly, 3.4.1 may be supplemented by a second such relationship if the efflux becomes supersonic and two acoustic characteristics intercept the boundary from the interior. We note that in such a case, therefore, p and u are completely determined as the solution of the two linear equations. On the other hand, if the state corresponds to a supersonic influx, then 3.4.1 may not be used at all; acoustic waves no longer reach the boundary from the interior of the flow. All these possibilities will be exercised in the subsequent discussion of the physical boundary conditions.

Our comments thus far have pertained specifically to the gas. Perfectly analogous considerations apply to the solid propellant. For future reference we state the characteristic forms as

$$\sigma = \alpha_p u_p + \beta_p \qquad 3.4.3$$

corresponding to a member of Γ_p^+ or Γ_p^- and

$$\rho_p = \theta_p \sigma + \phi_p \qquad 3.4.4$$

which corresponds to a member of Γ_p^o. In the present study we shall be concerned only with boundary conditions which express either impermeability to the solid propellant or subsonic efflux.

3.4.1 Breech (Closed or Open)

When the breech is closed, the physical boundary condition 2.5.1 yields u = 0 directly. Then the pressure follows directly from a condition of compatibility in the form 3.4.1 and the density from 3.4.2. The solution is therefore complete.

If the breech is open and the efflux is supersonic, then as noted above, p and u follow from the two relations of the form 3.4.1 corresponding to the two acoustic characteristic intersections. The density follows from 3.4.2. Before the solution can be accepted it is necessary to test that the mass flux does not exceed $\dot{m}_*$ given by equation 2.5.2. If the value $\dot{m}_*$ is exceeded, or if the flow is subsonic, the efflux is required to be as given by 2.5.2 and the condition of compatibility on the characteristic Γ_g^+ is not applicable. The boundary values are found iteratively, using a midpoint search.

3.4.2 Projectile Base and Muzzle Following Projectile Exit

At the projectile base, whether we are concerned with the continuum response of the solid propellant or with the behavior of the gas following burnout, the condition 2.5.5 or 2.5.6 yields the velocity. Then the pressure follows from 3.4.1 or 3.4.3, as the case may be, and the density from 3.4.2 or 3.4.4. The projectile velocity $\dot{X}_p$ is always updated prior to the determination of the boundary values.

The analysis of the boundary values at the muzzle following expulsion of the projectile is completely analogous to that pertaining to the open breech.

3.4.3 Gas/Propellant Interface

We consider several possible cases in accordance with the manner of specification of the burn rate. The discussion given for each case is predicated on the assumption that the solid propellant is treated as rigid so that its density is known as a constant value and its velocity, equal to that of the projectile, is given according to an update of the equation of motion 2.3.4. Therefore, we preface the analysis of each case by commenting on the consequences of a continuum representation of the solid propellant. In the event that the propellant is to be treated as a continuum we are required to satisfy two conditions of compatibility of the form 3.4.3 and 3.4.4. This is accomplished iteratively as follows.

A trial value for u_p is proposed, based on current storage for the boundary values on the unreacted side of the interface. Compatible values σ and ρ_p are deduced from 3.4.3 and 3.4.4. The values of u_p and ρ_p are used together with the remaining boundary conditions to determine consistent values for the state of the gas and also the pressure σ'. The latter value will not, in general be equal to σ. Accordingly, the proposed value of u_p is modified by an amount $\Delta u_p = (\sigma' - \sigma)/\alpha_p$, σ is

replaced by the now compatible value σ' and ρ_p is similarly adjusted. This procedure is continued until the pressure σ changes by an amount less than 6.9 Pa.

Branching from one condition to another is discussed in section 3.5.

3.4.3.1 Propellant Unreacting

This case may arise if, for example, an ideal burn rate is specified to yield a value of pressure on the unreacted side which is less than the current pressure on the reacted side. The ideal rate would therefore be negative and is replaced by the condition that the propellant is not burning. Given u_p, the value u is known at once as $u = u_p$ and the analysis is identical with that following burnout.

3.4.3.2 Measured Burn Rate (With Subsonic Reactants)

Substitution of 2.5.7 into 2.5.8 yields

$$\frac{\sigma}{\rho_p} = \frac{p}{\rho_p} + \frac{r^2}{g_o}\left[\frac{\rho_p}{\rho} - 1\right] \qquad 3.4.5$$

Similarly, substitution of 2.5.7 into 2.5.9 yields

$$\frac{\sigma}{\rho_p} = e + \frac{p}{\rho} - e_p + \frac{r^2}{2g_o}\left[\left(\frac{\rho_p}{\rho}\right)^2 - 1\right] \qquad 3.4.6$$

Then, combining 3.4.5 and 3.4.6 to eliminate σ and making use of the equation of state 2.4.1 to eliminate e we have

$$\frac{p}{\rho_p}\left[\frac{\gamma}{\gamma - 1}\frac{\rho_p}{\rho} - 1 - \frac{b\rho_p}{\gamma - 1}\right] = e_p - \frac{r^2}{2g_o}\left[\frac{\rho_p}{\rho} - 1\right]^2 \qquad 3.4.7$$

Furthermore, as the reactants are presumed subsonic, we may combine 2.5.7 with the condition of compatibility 3.4.1 to obtain

$$\frac{\rho_p}{\rho} = \frac{u_p + r - (p - \beta)/\alpha}{r} = \Psi(p) \qquad 3.4.8$$

Then 3.4.8 may be used to permit the interpretation of 3.4.7 as a function of p alone, namely

$$\Phi(p) = \frac{p}{\rho_p}\left[\frac{\gamma}{\gamma - 1}\Psi(p) - 1 - \frac{b\rho_p}{\gamma - 1}\right] - e_p + \frac{r^2}{2g_o}\left[\Psi(p) - 1\right]^2 = 0 \qquad 3.4.9$$

Equation 3.4.9 may be solved iteratively using Newton's method according to which the trial value p is replaced by the better approximation $p - \Phi(p)/\Phi'(p)$. We note the following derivatives which are required in the iterative process.

$$\Phi'(p) = \frac{1}{\rho_p}\left[\frac{\gamma}{\gamma - 1}\Psi(p) - 1 - \frac{b\rho_p}{\gamma - 1}\right] - \frac{p}{\rho_p}\frac{\gamma}{\gamma - 1}\Psi'(p) \qquad 3.4.10$$

$$+ \frac{r(p)r'(p)}{g_o}[\Psi(p) - 1]^2 + \frac{r^2(p)}{g_o}[\Psi(p) - 1]\Psi'(p)$$

$$\Psi'(p) = r^{-2}(p)\{r(p)[r'(p) - 1/\alpha] - [u_p + r(p) - (p - \beta)/\alpha]\, r'(p)\} \qquad 3.4.11$$

Once p is determined the remaining boundary values follow from the chain of calculations:

$$u = (p - \beta)/\alpha \qquad 3.4.12$$

$$r = r(p) \qquad 3.4.13$$

$$\rho = \frac{r\rho_p}{u_p + r - u} \qquad 3.4.14$$

$$\sigma = p + \frac{\rho_p r^2}{g_o}\left(\frac{\rho_p}{\rho} - 1\right) \qquad 3.4.15$$

$$e = \frac{p(1 - b\rho)}{(\gamma - 1)\rho} \qquad 3.4.16$$

As noted in the preamble to this section, if u_p was a trial value, the quantity σ given by 3.4.15 must be compared with the value which is compatible with u_p according to 3.4.3. If they differ, u_p must be adjusted appropriately and the boundary values redetermined.

3.4.3.3 Ideal Burning of Langweiler (Subsonic or Supersonic Reactants)

In this case the determination of the boundary values for the gas is almost trivial since u = 0 and the pressure p is equal to the constant initial value. Only the density at the boundary, and hence the internal energy, vary with time. When u = 0, 2.5.7 yields

$$\frac{\rho_p}{\rho} = \frac{u_p}{r} + 1 \qquad 3.4.17$$

Substituting this result into 3.4.7 and rearranging yields the ideal rate of burning as a function of the instantaneous velocity of the propellant.

$$r = \left\{\frac{\frac{\gamma}{\gamma - 1}\frac{p}{\rho_p}}{e_p + \frac{bp}{\gamma - 1} - \frac{p}{(\gamma - 1)\rho_p} - \frac{u_p^2}{2g_o}}\right\} u_p \qquad 3.4.18$$

Given r, ρ follows from 3.4.17 whereupon σ and e are determined by 3.4.15 and 3.4.16 respectively.

3.4.3.4 Prespecified Value of Pressure on Unreacted Side or of Acceleration of Projectile (With Subsonic Reactants)

As discussed in section 2.5.3 we assume that both of these conditions amount to the same thing, namely the specification of σ. Substitution of 3.4.5 into 3.4.6 to eliminate r yields, after rearrangement

$$\frac{\rho}{\rho_p} = \frac{\frac{p}{\rho_p}\frac{\gamma}{\gamma - 1} - \frac{p - \sigma}{2\rho_p}}{e_p - \frac{p - \sigma}{2\rho_p} + \frac{p}{\rho_p}\left(1 + \frac{b\rho_p}{\gamma - 1}\right)} \qquad 3.4.19$$

By combining 3.4.19 with 3.4.8 we may deduce

$$r = \frac{[u_p - \frac{p - \beta}{\alpha}][\sigma + \frac{\gamma + 1}{\gamma - 1}p]}{2\rho_p[e_p + \frac{bp}{\gamma - 1} - \frac{p}{\rho_p(\gamma - 1)}]} \qquad 3.4.20$$

This may be viewed as an effective or ideal burn rate equation yielding $r = r(p)$. Then the method of section 3.4.3.2 may be followed with 3.4.20 in place of the measured burn rate law.

3.4.3.5 Predetermined Mach Number of Reactants (Subsonic)

We may state the condition in the form

$$M = \frac{\dot{x}_p - u}{\sqrt{\frac{\gamma g_o p}{\rho(1 - b\rho)}}} \qquad 3.4.21$$

where $M < 1$ is the predetermined value of the Mach number. In view of 2.5.7 this may be solved to yield

$$\frac{\rho_p}{\rho} = \rho_p\left[\, b + \frac{\gamma g_o p M^2}{\rho_p^{\,2} r^2}\,\right] \qquad 3.4.22$$

Moreover, combining 3.4.22 with 3.4.8 we may deduce an effective burn rate law, $r = r(p)$

$$r = \frac{\frac{p - \beta}{\alpha} - u_p + \left\{\left(\frac{p - \beta}{\alpha} - u_p\right)^2 + \frac{4(1 - b\rho_p)}{\rho_p}\gamma g_o p M^2\right\}^{1/2}}{2(1 - b\rho_p)} \qquad 3.4.23$$

Then the analysis of 3.4.3.2 applies with r defined by 3.4.23.

3.4.3.6 Prespecified Value of Pressure on Unreacted Side or of Acceleration of Projectile (With Supersonic Reactants)

In this case 3.4.22, which expresses the additional requirement that the Mach number $M > 1$ of the reactants be given, may be combined with 3.4.5 to yield

$$r(p) = \left[\frac{g_o}{\rho_p} \frac{\sigma - (1 + \gamma M^2)p}{b\rho_p - 1} \right]^{1/2} \qquad 3.4.24$$

as an effective burn rate law. However, in making use of the analysis of section 3.4.3.2 it is necessary to use not only 3.4.24 to define $r(p)$ but also 3.4.22 to define $\Psi(p)$. The expression 3.4.8 incorporates the assumption that u and p are compatible with the interior flow of the gas which is no longer true in this case. Moreover, following the determination of p, it is necessary to deduce ρ from 3.4.22 whereupon u follows from 3.4.14.

3.5 Additional Considerations

We conclude this chapter by noting some additional considerations relating to the method of solution.

3.5.1 Choice of Time Step

As is well known[17], explicit finite difference schemes demand, for their numerical stability, that the time step be suitably constrained. We constrain Δt according to the Courant-Friedrichs-Lewy (C-F-L) condition

$$\Delta t \leq \frac{x_p \Delta\zeta}{\max[|u - \eta| + c]} \qquad 3.5.1$$

where $\Delta\zeta$ is the nondimensional mesh spacing in the gas, and by a similar relation expressed in terms of the properties of the propellant when it, too, is treated as a continuum.

The maximum allowable value of Δt thus defined is further divided by a user-supplied safety factor, SAFE, which must be equal to at least one.

3.5.2 Treatment at Burnout

If the combustion zone is represented as a discontinuity of infinitesimal thickness, it follows that the boundary value of the velocity of the gas will undergo a sudden jump as burnout occurs. The flow which will have been represented as blowing away from the base of the projectile will now be required to follow it. If the Mach number of the

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[17] *Richtmyer, R. D. and Morton, K. W. "Difference Methods for Initial Value Problems" 2nd Ed. John Wiley, New York 1967*

reactants, just prior to the instant of burnout, was comparable to unity, the sudden change in velocity will be accompanied by a sharp drop in pressure. A strong rarefaction will be formed to communicate the change in the boundary condition to the column of gas. Indeed, the sudden drop in pressure may be sufficiently large as to strain the numerical method of solution. Therefore, we employ an analytical solution to describe the boundary values for a short period after burnout. The analysis also provides a simple exact solution which may be used to benchmark the accuracy of the program, as we discuss in chapter 4.0. We therefore deduce the solution in some detail before commenting on its application to the determination of the boundary values in the computer program.

Consider the motion of a piston propelled by a semi-infinite column of gas which is initially at rest and has uniform properties. At any subsequent time, the gas may be divided into two regions, one still quiescent and uniform, and the other undergoing expansion to follow the projectile. A flow of this type is called a simple wave [8]. If we assume the piston to be moving to the right, it follows that all characteristics of the family Γ_g^+ emanate from the uniform region so that their corresponding condition of compatibility is impressed uniformly on the flow. We therefore have, throughout the expansion region

$$u + \int c\frac{dp}{\rho} = \text{constant} \qquad 3.5.2$$

Using the covolume equation of state to perform the thermodynamic integral and denoting the properties of the quiescent region by a subscript 1 we have

$$u + \frac{2}{\gamma - 1}(1 - b\rho)c = \frac{2}{\gamma - 1}(1 - b\rho_1)c_1 \qquad 3.5.3$$

This relation applies, in particular, at the base of the piston where $u = \dot{X}_p$. Moreover, the isentropic nature of the flow implies

$$\frac{p}{p_1} = \left[\frac{(1 - b\rho)c}{(1 - b\rho_1)c_1}\right]^{\frac{2\gamma}{\gamma-1}} \qquad 3.5.4$$

Combining 3.5.3 and 3.5.4 yields the base pressure as a function of the velocity of the piston

$$p = p_1\left[1 - \frac{\gamma - 1}{2}\frac{\dot{X}_p}{(1 - b\rho_1)c_1}\right]^{\frac{2\gamma}{\gamma-1}} \qquad 3.5.5$$

From the equation of motion of the projectile it is a simple matter to deduce the following.

$$\dot{X}_p = \frac{2c_1'}{\gamma - 1}\left\{1 - \left[\left(1 - \frac{\gamma - 1}{2}\frac{\dot{X}_{p_o}}{c_1'}\right)^{-\frac{\gamma+1}{\gamma-1}} + \frac{\gamma + 1}{2c_1'}\frac{A_b g_o p_1}{M_p}(t - t_o)\right]^{-\frac{\gamma-1}{\gamma+1}}\right\} \qquad 3.5.6$$

$$X_p = X_{p_o} + \frac{2c_1'}{\gamma - 1}\left\{(t - t_o) + \frac{M_p c_1'}{A_b g_o p_1}\left[\left(1 - \frac{\gamma - 1}{2}\frac{\dot{X}_{p_o}}{c_1'}\right)^{-\frac{2}{\gamma-1}}\right.\right.$$

$$\left.\left. - \left[\left(1 - \frac{\gamma - 1}{2}\frac{\dot{X}_{p_o}}{c_1'}\right)^{-\frac{\gamma+1}{\gamma-1}} + \frac{\gamma + 1}{2c_1'}\frac{A_b g_o p_1}{M_p}(t - t_o)\right]^{\frac{2}{\gamma+1}}\right]\right\} \qquad 3.5.7$$

It should be noted that we have taken the initial instant to be t_o and we have denoted the initial position and velocity of the piston by X_{p_o} and $\dot{X}_{p_o}$ respectively. Also, we have used

$$c_1' = (1 - b\rho_1)c_1 \qquad 3.5.8$$

The relations 3.5.6 and 3.5.7 with $\dot{X}_{p_o} = 0$ provide an exact solution which may be used to benchmark the accuracy of the computer program. Since the code must be exercised with a finite column of gas, the comparison will only be valid until such a time as the reflection of the expansion front from the breech overtakes the piston or projectile.

When the ideal combustion of Langweiler has been in effect up to the instant of burnout, the gas is quiescent as assumed above. Then 3.5.5 may be used to determine the boundary value of pressure. We note that if, at the instant of burnout we have a near sonic condition $\dot{X}_p = c_1'$ and $\gamma = 1.2$, then $p/p_1 = 0.282$ so that the pressure drops instantaneously to 28% of its value prior to burnout. This implies a sharp drop in the propulsive capacity of the gas which is exacerbated by the fact that prior to burnout, propulsion was due not just to p_1 but also to the contribution of thrust.

When the combustion is due to other than the ideal model of Langweiler, the state of the gas adjacent to the projectile at burnout is, in general, neither uniform nor quiescent. We nevertheless use 3.5.5, where p_1 and c_1 are predicated on the conditions at the boundary just prior to burnout, and we replace $\dot{X}_p$ by $\dot{X}_p - u_1$ where u_1 is the velocity of the gas just prior to burnout.

In all cases, equation 3.5.5 is used to determine the boundary value of pressure for five integration steps following burnout. Thereafter, the previously described numerical algorithm is used.

3.5.3 Change of Representation of Solid Propellant

As the solid propellant is consumed, it will reach a point at which it is too short to be treated as a continuum in accordance with the constraint $(X_p - x_p)\Delta\zeta \geq$ DXMIN on the mesh spacing $\Delta\zeta$. At this point, if a continuum representation had been elected, an internal switch is reset and the calculation concludes with the propellant treated as rigid. A value of the density is computed from a knowledge of the remaining mass of propellant and its length. Friction between the propellant and the wall is not treated in cases in which the propellant is initially taken

to be rigid. When a transformation from a continuum to a rigid representation is made, near burnout, the resistance per unit bearing area due to friction is frozen at the value it had at the time of transition.

3.5.4 Treatment of Friction Between Propellant and Tube Wall

The friction term, when taken to be proportional to pressure in accordance with equation 2.4.9, with values of $\mu_w \gtrsim 0.1$, becomes very large and tends to create numerical wiggles in the solution. A number of different schemes were attempted in order to resolve this problem. Indeed, the use of centered differencing rather than the alternating scheme of MacCormack, in the integration of the momentum equation of the propellant, was predicated on the desire to express as accurately as possible the competition between $\partial\sigma/\partial x$ and f_w at both predictor and corrector levels. Possibly, calculations with values of μ_w as large as 0.1 will not be of interest since the ballistic loss will be seen to be significant, at least in the sample cases described in chapter 4.0.

Nevertheless, to stabilize properly such solutions we have smoothed the term f_w. In the numerical evaluation of equation 2.4.9 the pressure is expressed as $(p_{j-1} + p_{j+2} + 2p_j)/4$ at internal mesh points and as an average of the boundary value with its neighbor at boundary points. Also, the expression for f_w is made implicit in the pressure at the boundaries when using the characteristic forms. That is to say the term f_w is multiplied by p_1/p_2 in the nomenclature of section 3.4 prior to making use of the characteristic form 3.4.3.

3.5.5 Branching of Conditions at the Gas/Propellant Interface

As we have noted previously, the program is structured so as to allow the combustion model to vary in nature as the ballistic cycle unfolds. Our purpose in this concluding section is to state the rules according to which the internal branching has been structured. The following are the combinations of constraints which may be involved during a given cycle.

(a) Measured Burn Rate/Mach Number Limit

If measured burn data are supplied, a search is made for boundary values based on these data. The values so found are then tested to ensure that the Mach number of the reactants does not exceed the limit M, provided $M > 0$. If M is exceeded or, if the search for boundary values based on the measured burn rate was unsuccessful, branching occurs to determine values which yield the specified value of M.

This combination is limited to subsonic combustion, that is, $M < 1$. If the measured data yield a Mach number greater than unity and the user has specified $M = 0$ or $M \geq 1$, the calculation terminates with an error message and the program proceeds to the next case.

(b) Langweiler Combustion/Mach Number Limit

The Langweiler ideal combustion model will yield boundary values

which are only limited thermodynamically by the covolume limit $\rho = 1/b$ at which point the reactants are at zero temperature. If a value $0 < M < 1$ has been furnished by the user, the boundary values according to the Langweiler model are tested to ensure that the Mach number of the reactants does not exceed M. If it does, branching will occur to yield the indicated Mach number. It should be noted that while the Langweiler model will admit supersonic combustion, the branch to the Mach number limit will be admitted only if $M < 1$.

(c) Prespecified Propellant Pressure or Acceleration/Mach Number Limit

It is simple to branch between a constraint on the pressure on the unreacted side of the interface and a constraint on the acceleration, as both of these constraints can be expressed in terms of the pressure on the unreacted side of the interface. A search for suitable boundary values is undertaken. When they are found, the Mach number is compared with the input limit M, provided $M > 0$. If $M < 1$ and the limit has been exceeded, branching to the Mach number limitation will occur. If the computed Mach number and the input limit M are both greater than or equal to one, branching occurs to the supersonic combustion process to satisfy both M and the required value of pressure simultaneously. The flow is no longer required to be compatible with the state of the gas.

If the original search for subsonic boundary values to yield the designated pressure on the unreacted side was unsuccessful, and $M > 0$, branching will occur to the Mach number limited processes, using M alone if $M < 1$ and using both the designated pressure and M if $M \geq 1$.

There is one further branch to be considered in this case. If the value of the indicated pressure on the unreacted side is less than or equal to the current pressure of the gas, the reaction is assumed to terminate since the regression rate would otherwise be required to be negative.

4.0 SOME NUMERICAL RESULTS

The model which we have described in chapter 2.0 admits a large variety of possible cases. A systematic exploitation of the model capacity is beyond the scope of the present study. The solutions which we discuss in the present chapter are intended principally to demonstrate the operability of the computer program and to provide an assessment of the magnitude of the errors associated with the method of solution. However, we also comment briefly on the potential magnitude of some of the losses associated with the end-burning traveling charge. First, in section 4.1, we consider a case for which an exact solution is available in order to provide an absolute benchmark of accuracy. Second, in section 4.2, we discuss a nominal traveling charge configuration, paying attention to properties of mesh indifference and global conservation of mass and energy. Finally, in section 4.3, we comment on the losses due to friction, heat transfer to the tube, and the pressure of shocked air in front of the projectile.

4.1 Comparison with an Exact Solution

In section 3.5 we presented an exact solution for the motion of a piston propelled by a semi-infinite column of initially quiescent, uniform gas. While the present model can only be used in the context of a finite tube, its predictions may be compared with the exact solution for that period of time prior to the interaction of the expansion wave with the finite boundary. Such a comparison is made in Table 4.1. The solutions were based on values D = 4.0 cm, γ = 1.239, M = 25.0 gm-mol/gm, b = 1.06 cm^3/gm, M_p = 160 gm, an initial pressure of 551.6 MPa and an initial temperature of 3271°K. The numerical solutions were deduced for a tube of length 254 cms. Inasmuch as the speed of sound in the quiescent gas is 1.78 km/sec, it follows that equations 3.5.5 and 3.5.6 may be used to describe the pressure at the base of the piston and the piston velocity for a period of at least 1.4 msec, after which time the wave reflection from the finite boundary begins to invalidate the exact solution.

Table 4.1 Comparison of Numerical Solutions with an Exact Solution

Time (msec)	Base Pressure (MPa)			Piston Velocity (km/sec)		
	Exact	21 pts	100 pts	Exact	21 pts	100 pts
0.2	281.8	284.9	281.9	0.610	0.612	0.610
0.4	186.1	186.0	186.1	0.967	0.970	0.967
0.6	137.7	137.8	137.7	1.217	1.221	1.217
0.8	108.7	108.9	108.7	1.409	1.412	1.409
1.0	89.5	89.5	89.5	1.564	1.567	1.564

The table provides a comparison between the exact solution and numerical solutions generated with 21 and with 100 mesh points. The agreement is seen to be very good. Even with 21 mesh points, the base pressure is predicted to within approximately 1% of the exact value at 0.2 msec and

to within less than 0.1% at later times. With 100 points the error is less than 0.03% at 0.2 msec. As expected, the results pertaining to the piston velocity are similar. The results are indicative not only of good absolute accuracy but also of mesh indifference. It is worth noting that the expansion is really very strong. Evidently, due to the large initial pressure and the relatively low piston mass, the pressure drops to roughly 50% of its initial value by 0.2 msec.

4.2 A Nominal Traveling Charge Configuration

To illustrate further the operability of the computer program we now present a solution for a nominal end-burning traveling charge configuration. The problem of interest corresponds to a 40 mm tube with 100 calibers of projectile travel. An ideal burning law is assumed in which the pressure on the unreacted side of the gas/propellant interface is required to be 700 MPa provided that the Mach number of the reactants does not exceed 0.999. In accordance with the logic for branching which we have discussed in the previous chapter, a solution yielding the requisite pressure will be sought at each stage of the calculation. If the Mach number limitation is exceeded, branching will occur to yield values satisfying the limiting value of the Mach number. As the input value is less than unity, the reactants will be required to satisfy a condition of mechanical compatibility with the column of gas.

We first solve this problem treating the propellant as rigid. Three solutions are generated using a maximum of 21, 41, and 81 mesh points and minimum spacings of 0.508, 0.254 and 0.127 cms respectively. Some aspects of these three solutions are compared in order to permit an assessment of the mesh indifference of the predictions. We also consider the degree to which mass and energy are conserved globally in these three calculations as a further indication of the magnitude of the numerical errors of integration. Subsequently, we present some details of a fourth calculation of the same problem based on a continuum representation of the propellant. In this last case we illustrate the solution by means of plots of the pressure and velocity distributions at various times.

The data base for the nominal problem is contained in Table 4.2. The values for the wave speed in the propellant only pertain, of course, to the case in which continuum behavior is considered.

Table 4.2 Data Base for Nominal Traveling Charge Problem

Tube Diameter	4.0 cm
Maximum Projectile Travel	4.0 m
Projectile Mass	160 gm
Propellant Chemical Energy	4450 J/gm
Covolume	1.05 cm^3/gm
Molecular Weight	23.253 gmol/gm
Density	1.66 gm/cc
Ratio of Specific Heats	1.2414 (-)
Compressive Wave Speed	1.07 km/sec

Unloading Wave Speed*	0 m/sec
Initial position of rear propellant face relative to breech	2.54 cm
Initial Pressure	700 MPa
Maximum allowable pressure on unreacted side of gas/propellant interface	700 MPa
Maximum allowable reactant Mach number	0.999
Charge-to-projectile mass ratio	3.54
Loading density	1.2 gm/cc

*Code action is to treat solid propellant constitutive law as reversible in case of continuum representation.

In Table 4.3 we compare the values of projectile velocity and breech and base pressure in the gas for the three calculations based on a representation of the propellant as rigid. The base pressure corresponds to the state of the reactants at the gas/propellant interface.

Table 4.3 Dependence of Nominal Solution on Mesh

Time (msec)	Projectile Velocity (km/sec)			Breech Pressure (MPa)			Base Pressure		
	21 pts	41 pts	81 pts	21 pts	41 pts	81 pts	21 pts	41 pts	81 pts
0.2	0.249	0.249	0.249	664.4	664.4	664.4	663.5	663.5	663.5
0.4	0.511	0.511	0.511	583.1	583.1	583.1	579.1	579.1	579.1
0.6	0.804	0.804	0.804	492.6	492.6	492.6	481.1	481.2	481.2
0.8	1.144	1.144	1.144	410.0	410.0	410.0	384.9	384.9	385.0
1.0	1.559	1.559	1.559	340.1	340.1	340.1	291.6	291.6	291.6
1.6	2.456	2.458	2.460	199.5	199.8	199.8	84.8	85.1	85.3
2.0	2.821	2.825	2.828	140.7	142.1	142.9	52.4	52.6	52.7
2.34*	3.068	3.073	3.076	103.8	102.9	102.1	38.0	38.1	38.2

*Muzzle Exit

The quantities of ballistic interest are the maximum pressures and the muzzle velocity. These do not exhibit a mesh dependence beyond 0.2% as we pass from 21 to 41 points or beyond 0.1% as we pass from 41 to 81 points. The greatest numerical error seems to be associated with the breech pressure at muzzle exit which exhibits a mesh dependence of approximately 0.7%. As the value of the breech pressure at muzzle exit is not normally of ballistic interest, it appears from Table 4.3 that 41 mesh points are sufficient in simulations of this type.

Next, in Table 4.4, we present the histories of the defects in total mass and energy at the same set of times, for the three calculations. Since there are no losses in the physical problem, both mass and energy should be conserved on a global basis. As the present numerical scheme does not automatically assure such a global conservation, the mass and energy defects provide an indication of the magnitude of the errors of numerical integration. The total mass is computed using a trapezoidal rule. The result is subtracted from the initial value and the defect is then expressed as a percentage of the original value. The energy defect is computed similarly.

Table 4.4 Global Mass and Energy Defects in Nominal Problem

Time (msec)	Mass Defect (%) 21 pts	41 pts	81 pts	Energy Defect (%) 21 pts	41 pts	81 pts
0.2	-0.01	0.00	0.00	0.00	0.00	0.00
0.4	0.00	0.00	0.00	0.00	0.00	0.00
0.6	0.00	0.00	0.00	0.00	0.00	0.00
0.8	0.02	0.00	0.00	0.00	0.00	0.00
1.0	0.07	0.01	0.00	0.00	0.00	0.00
1.6	0.26	0.06	0.01	-0.01	0.00	0.00
2.0 *	0.38	0.10	0.02	0.02	0.01	0.00
2.34*	0.45	0.11	0.02	0.04	0.02	0.01

*Muzzle Exit

The results of Table 4.4 essentially confirm those of Table 4.3. If the maximum defect is taken as an indicator of the probable error of any given ballistic prediction, it follows from Table 4.4 that errors of the order of 0.5% are to be expected if one uses a maximum 21 mesh points and of the order of 0.1% if one uses 41 points.

We turn now to a discussion of a solution for the nominal problem based on a continuum representation of the propellant. The solution is illustrated by the distributions of pressure and velocity shown in figures 4.1 through 4.8.

It should be noted that the solid propellant is assumed to have a nonlinear, but reversible, equation of state. A maximum of 31 mesh points is allowed with a minimum spacing of 0.508 cms. The maximum time step as computed from the Courant-Friedrichs-Lewy condition is divided by two so that the maximum Courant number is, by definition, 0.5.

The initial distributions of pressure and velocity are shown in figure 4.1. As the initial pressure is equal to the maximum value required on the unreacted side of the interface, the burn rate is initially zero according to the ideal law. Both the gas and the solid propellant are at rest and pressurized uniformly to the initial value. Friction between the propellant and the tube wall is not considered. We will

comment on the influence of the loss terms in the next section of this chapter.

Because of the high pressure exerted on the projectile base, the projectile begins to accelerate and, analogously with the example of section 4.1, an expansion wave is propagated through the solid propellant. Figure 4.2 shows the conditions at 0.2 msec. The drop in pressure across the propellant is seen to be considerable. At this time, burning of the propellant has begun but is still rather weak since the gas pressure is still high. By 0.4 msec, as shown in figure 4.3, significant displacement of the projectile and traveling charge has occurred and the gas/propellant interface manifests itself quite clearly as a discontinuity in the distributions of both pressure and velocity.

Although the ideal combustion model is not that corresponding to the Langweiler process, it may be seen that the kinetic energy of the gas, as inferred from the velocity distribution, is still quite small at this stage. The same is true at 0.6 msec, figure 4.4, and even at 1.0 msec, figure 4.5, in which the propellant is seen to be approximately 50% burnt. The wave dynamics in the solid propellant should also be noted. In both figures 4.4 and 4.5, the pressure gradient in the solid propellant is seen to have reversed as a consequence of transient phenomena. Of equal interest, although not shown in the figures, is the fact that tensile stresses are predicted to occur at the interface between the propellant and the projectile base. However, undue significance should not be attached to this observation since the wave dynamics are associated to a large degree with the assumed highly pressurized initial condition.

The three concluding figures 4.6, 4.7 and 4.8 show the evolution of the solution until muzzle exit occurs. The propellant does not quite burn out in this example. It should be noted, in these last three figures, that branching to a Mach number limited burning process has occurred. The pressure at the base of the propellant is no longer 700 MPa but is, instead, roughly twice that in the gas. As the burn rate becomes limited by the Mach number constraint, the velocity profile of the gas develops and the gas does begin to store increasing amounts of kinetic energy. Nevertheless, it is easy to see, from figure 4.8, that the kinetic energy of the propellant is approximately 25% of what would be expected in a conventional charge. In the latter, the velocity of the gas at the projectile base would be equal to that of the projectile whereas in the present calculation it is seen to be approximately one-half of the projectile velocity.

In the latter figures there are still signs of the transient phenomena we noted previously in connection with the solid propellant. They are manifested here in the pressure distribution of the gas which has no mechanism for damping. Thus the variations in the ideal burn rate due to the interaction of traveling waves in the solid propellant with the gas/propellant interface have been preserved in the pressure field of the gas.

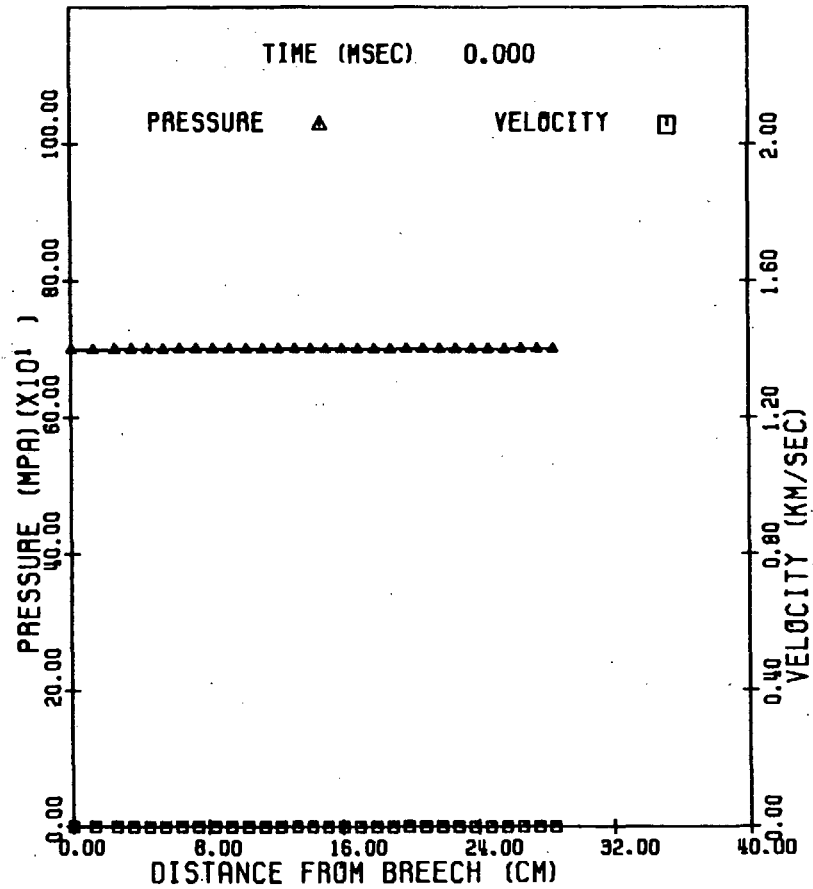

Figure 4.1 Distributions of pressure and velocity in nominal traveling charge problem at time 0.0 msec

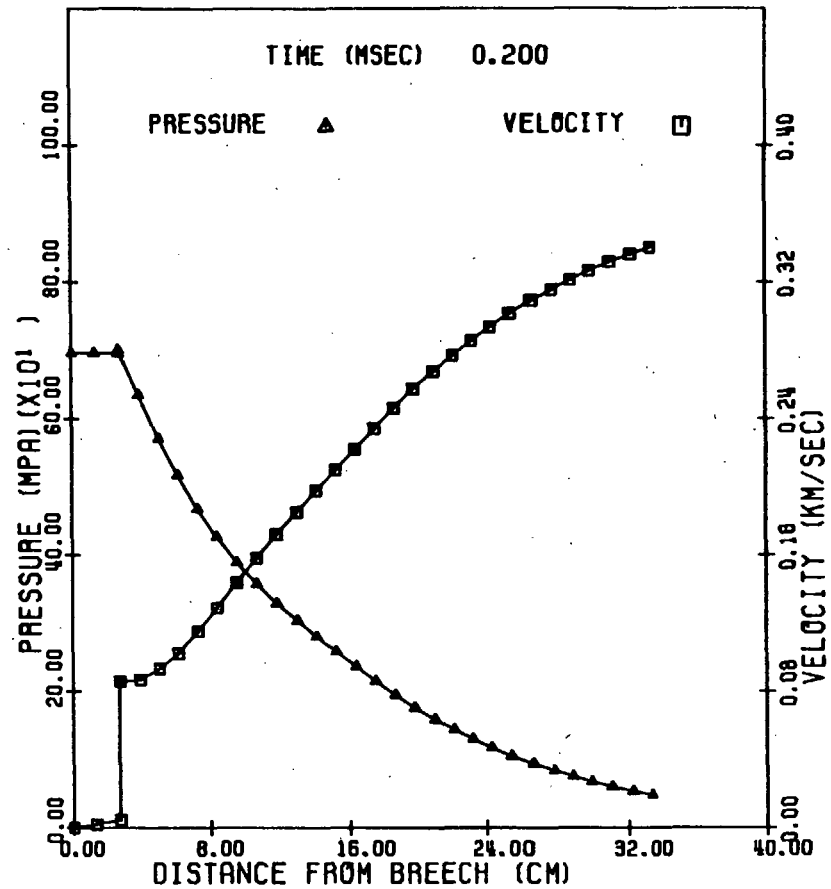

Figure 4.2 Distributions of pressure and velocity in nominal traveling charge problem at time 0.2 msec

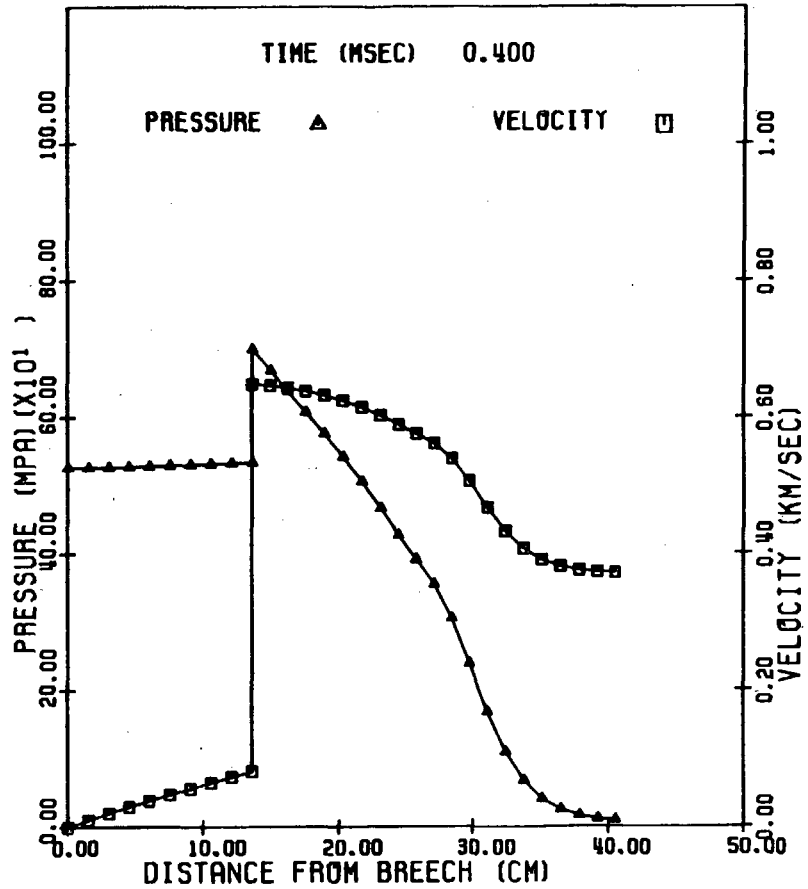

Figure 4.3 Distributions of pressure and velocity in nominal traveling charge problem at time 0.4 msec

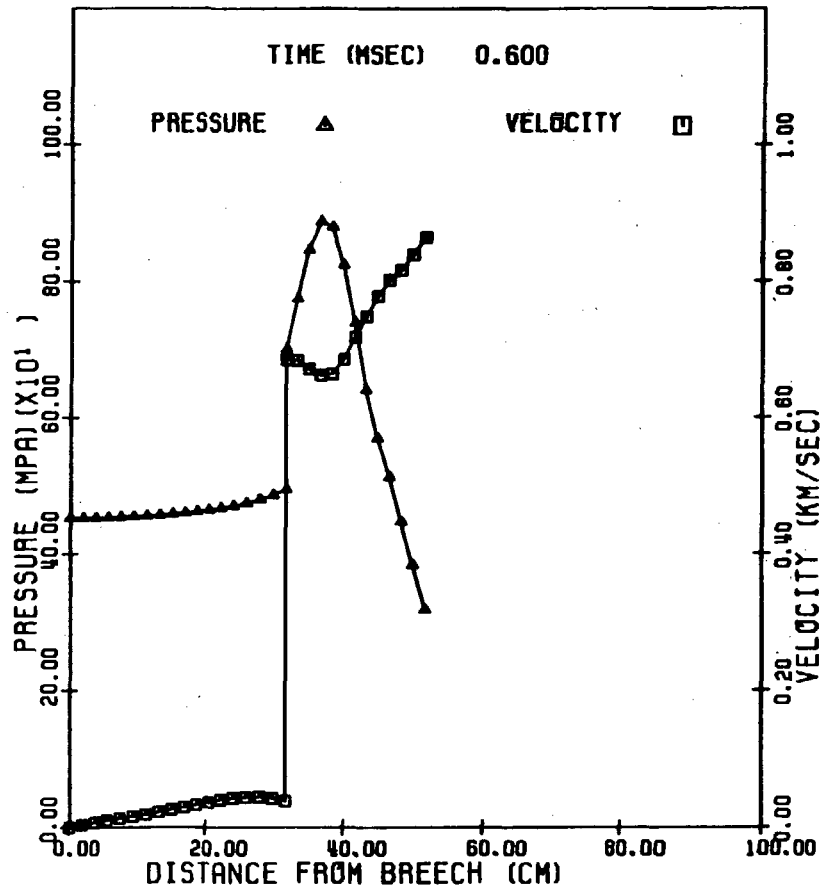

Figure 4.4 Distributions of pressure and velocity in nominal traveling charge problem at time 0.6 msec

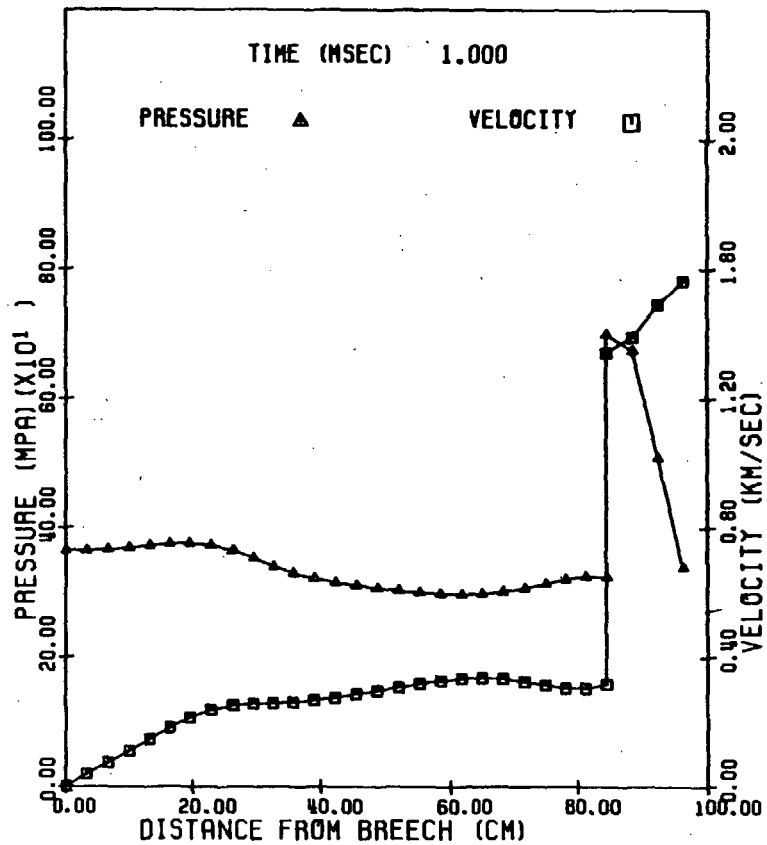

Figure 4.5 Distributions of pressure and velocity in nominal traveling charge problem at time 1.0 msec

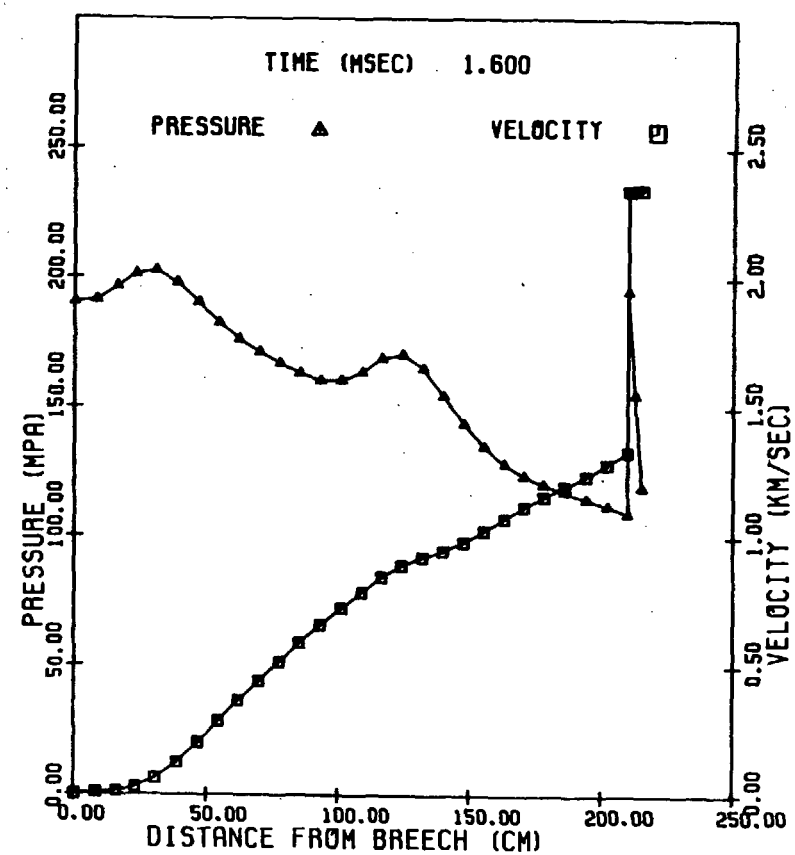

Figure 4.6 Distributions of pressure and velocity in nominal traveling charge problem at time 1.6 msec

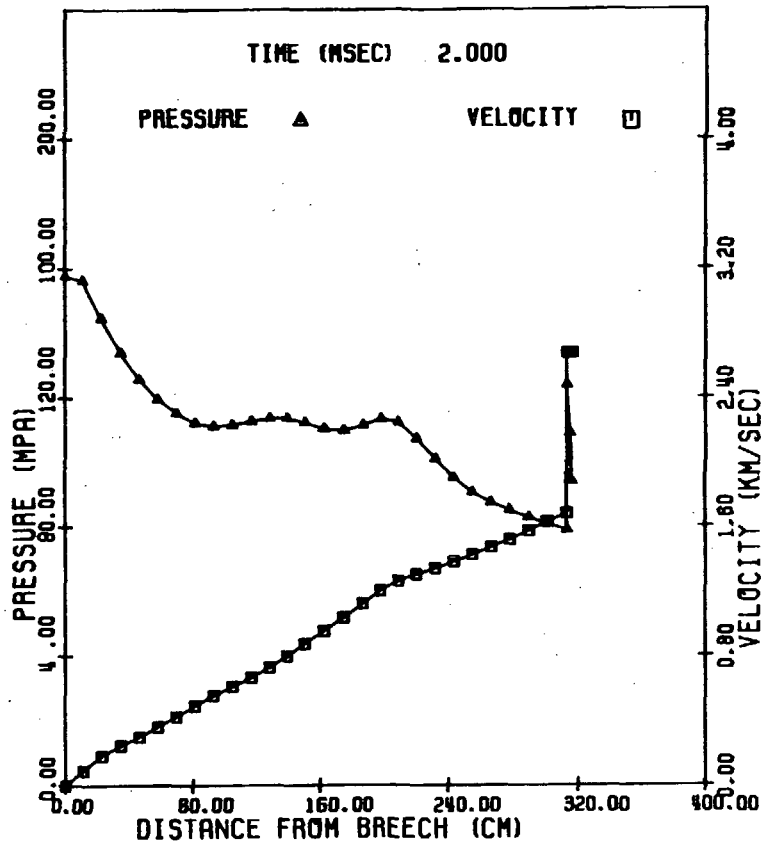

Figure 4.7 Distributions of pressure and velocity in nominal traveling charge problem at time 2.0 msec

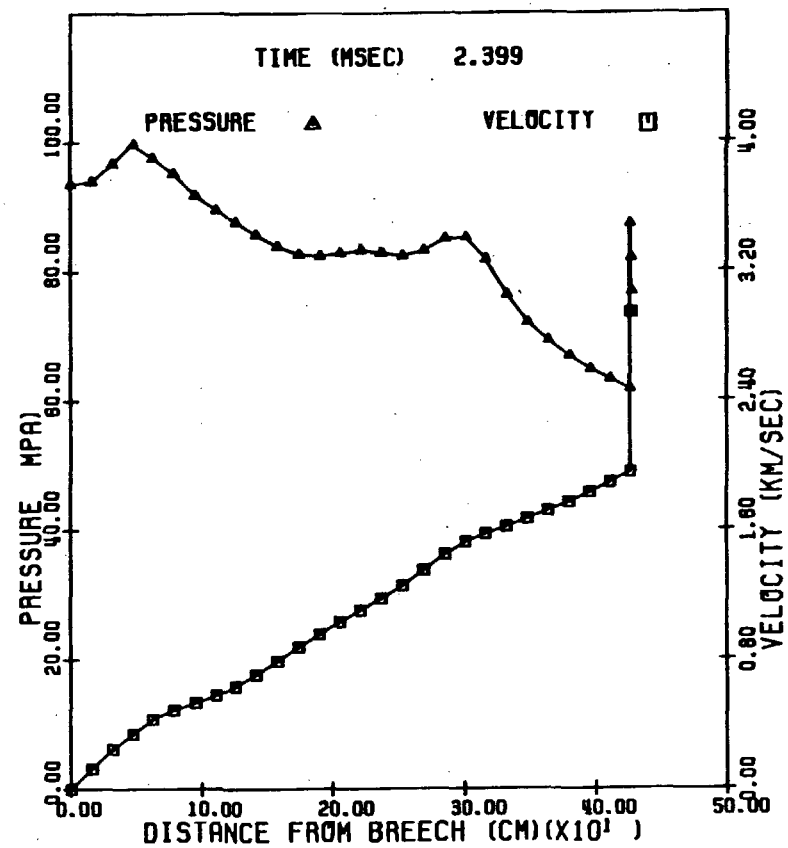

Figure 4.8 Distributions of pressure and velocity in nominal traveling charge problem at time 2.4 msec

4.3 Influence of Losses

To conclude, we summarize the results of calculations performed during the testing of the code, with a view to forming an assessment of the magnitudes of the losses to be expected in the traveling charge. These results are summarized in Table 4.5.

Table 4.5 Influence of Losses on Muzzle Velocity

Case	Comments	Muzzle Velocity (km/sec)
1	Nominal with rigid propellant	3.068
2	Propellant treated as a continuum	2.941
3	Propellant treated as a continuum with wall friction due to constant coefficient of friction equal to 0.1	2.194
4	Propellant treated as a continuum with wall friction coefficient given as following table of values Velocity (km/sec) / μ (-) 0. / 0.06 0.152 / 0.02 0.304 / 0.01	2.981
5	Propellant treated as a continuum with wall friction due to gas film bearing with $\mu_f = 1.79 \times 10^{-3}$gm/cm-sec and $\delta_f = 0.254$ mm	2.937
6	Rigid propellant and heat losses	2.997
7	Rigid propellant and resistance due to shocked air	3.025
8	Rigid propellant and constant resistance, due to obturator, of 3.45 MPa	3.059
9	Rigid propellant and resistance due to setback load on obturator of length 2.54 cms with forward mass of projectile equal to 160 gm and coefficient of friction given according to the following table of values Velocity (km/sec) / μ (-) 0. / 0.6 0.152 / 0.2 0.304 / 0.1	2.986

From this table it is evident that the only losses of significance are likely to be associated with friction between the propellant and the tube. A comparison of cases 2 and 3 shows that a friction coefficient of 0.1 simply cannot be tolerated. On the other hand, a mild coefficient of friction, such as that considered in case 4, may actually increase the muzzle velocity. This is a consequence of the failure of case 2 to yield burnout of the propellant whereas the friction in case 4 delays the projectile exit sufficiently for complete burnout to occur. The influence of the resistance in this case is therefore similar to that of a shot start pressure which, as is well known, often acts to increase the muzzle velocity in conventional ammunition.

5.0 CONCLUSIONS

The following conclusions are drawn in respect to the present study.

(1) A model of the end-burning traveling charge has been formulated, encoded, and demonstrated. The model is suitable for assessing the hydrodynamic and ideal combustion limits on the ballistic performance of traveling charge guns.

(2) Good agreement has been shown with an exact solution for a simplified case. Studies of mesh indifference and global conservation of mass and energy indicate that the numerical error associated with ballistic predictions of a nominal traveling charge configuration based on 41 mesh points is of the order of 0.1%-1.0%.

(3) A review of the arguments which deny the admissibility of a strong steady deflagration wave has revealed that this combustion limit is a manifestation of the well known steady flow process of choking by heat addition. It does not appear likely that this limit can be circumvented without sacrificing ballistic performance by the incorporation of condensed additives to control the effective flow area of the gas phase within the flame.

(4) Preliminary calculations have shown the importance of proper lubrication of the interface between the traveling charge and the tube. A coefficient of friction equal to 0.1 resulted in a loss of 30% of the muzzle velocity predicted without friction.

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Nomenclature

A_b	Cross sectional area of tube
A_*	Throat area of gas-permeable breech
a	Wave speed in solid propellant
a_1	Value of a at zero pressure
a_2	Value of a during unloading or reloading
α	Acceleration
B_1	Additive constant in measured burn rate law
B_2	Pre-exponential factor in measured burn rate law
b	Covolume of gas
C_{DB}	Discharge coefficient of gas-permeable breech
C_{DMUZ}	Discharge coefficient of muzzle
c	Isentropic sound speed in gas
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
D	Diameter of tube
e	Internal energy of gas
e_p	Chemical energy of propellant
F	Resistance to projectile motion
f_w	Force of friction exerted on wall by propellant
g_o	Constant used to reconcile units of measurement
h	Film coefficient
ℓ_b	Length of bearing section of obturator
M_b	Mass of projectile in front of midpoint of obturator
M_p	Mass of projectile
M	Molecular weight of gas
m_p	Mass of propellant
n	Burn rate exponent
Pr	Prandtl number
p	Pressure in gas
p_a	Pressure in shocked air ahead of projectile
p_{band}	Resistive pressure due to obturator
p_S	Shot start pressure
p_{ST}	Initial pressure of gas
p_o	Initial pressure of air ahead of projectile

Q_w	Coefficient in correlation for heat loss to tube
q_w	Heat loss to tube
Re_D	Reynolds number based on D
R	Gas constant
r	Regression rate of propellant
T	Gas temperature
T_w	Wall temperature
t	Time
u	Gas velocity
u_p	Propellant velocity
W	Charge to Projectile Mass Ratio
x	Axial coordinate
X_p	Position of projectile base relative to breech
x_p	Position of gas/propellant interface relative to breech

α	Coefficient in acoustic characteristic form for gas
α_p	Coefficient in acoustic characteristic form for propellant
β	Additive term in acoustic characteristic form for gas
β_p	Additive term in acoustic characteristic form for propellant
$\Gamma_g^o, \Gamma_g^{\pm}$	Gas-material, gas-acoustic characteristic lines
$\Gamma_p^o, \Gamma_p^{\pm}$	Propellant-material, propellant-acoustic characteristic lines
γ	Ratio of specific heats of gas
γ_a	Ratio of specific heats of air
Δt	Time step
$\Delta\zeta$	Spacewise mesh increment in computational plane
δ_f	Thickness of gas film used to lubricate propellant
ζ	Nondimensional axial coordinate in computational plane
η	Velocity of convective mesh
θ	Coefficient in material characteristic form for gas
θ_p	Coefficient in material characteristic form for propellant
κ	Thermal conductivity of gas
μ	Viscosity of gas

μ_f	Viscosity of lubricating film
μ_w	Coefficient of friction between propellant and tube wall
μ_{wb}	Coefficient of friction between obturator and tube wall
ν	Poisson ratio of projectile
ρ	Density of gas
ρ_p	Density of propellant
ρ_{p_o}	Value of ρ at zero pressure
σ	Pressure in propellant
σ_*	Value of σ on nominal loading curve

A dot over a quantity indicates a total derivative with respect to time.

Appendix A: On the Deflagration Wave with Supersonic Reactants

The purpose of this appendix is to note briefly the arguments which have been advanced to deny the possibility of a steady deflagration with supersonic reactants as perceived by an observer moving with the flame. These arguments are based on the details of the structure of the flame and have been established in the context of a single phase gas flow. Subsequently, we comment on the implications of a heterogeneous two-phase structure in regard to the possibility of supersonic flow. Finally, we note the implications of our findings as regards the traveling charge.

The theory of detonations and deflagrations, as it affects our present discussion, is treated fully in the monograph of Courant and Friedrichs[8]. From their development we shall abstract only those results which bear directly on the present discussion. In particular, we will note their treatment of the allowable states which may be reached in a steady exothermic flow without the restrictions imposed by considerations of the structure of the reaction zone. This discussion will provide the essential nomenclature and conceptual framework within which the basis for denial of the possibility of a deflagration with supersonic reactants can be understood. Following the presentation of this denial we will note the comments of other authors in relation to the single phase deflagration.

The basic constraints on the allowable steady state processes with properties observed at two stations, which we designate by subscripts 0 and 1 respectively, are the laws of conservation of mass, momentum and energy. We have already noted these in the present report. In order to keep the discussion of this appendix self-contained we restate them here and we introduce somewhat different nomenclature. The symbols used in this appendix will be defined as they are introduced.

Using p and ρ to denote pressure and density and letting v be the gas velocity in a frame of reference moving with the reaction zone we have

$$\rho_0 v_0 = \rho_1 v_1 = m \tag{A.1}$$

$$p_0 + \rho_0 v_0^2 = p_1 + \rho_1 v_1^2 \tag{A.2}$$

$$E^{(0)}(\tau_0, p_0) + p_0\tau_0 + \frac{1}{2} v_0^2 = E^{(1)}(\tau_1, p_1) + p_1\tau_1 + \frac{1}{2} v_1^2 \tag{A.3}$$

and where we have also introduced $\tau = 1/\rho$ and E is the internal energy, assumed to be given as a function of τ and p. The superscripts on the values of internal energy in (A.3) indicate that due to changes of composition, the functional form may vary from station 0 to station 1. The nomenclature used here is identical with that of Courant and Friedrichs. We will adopt the convention that the gas flows from station 0 to station 1.

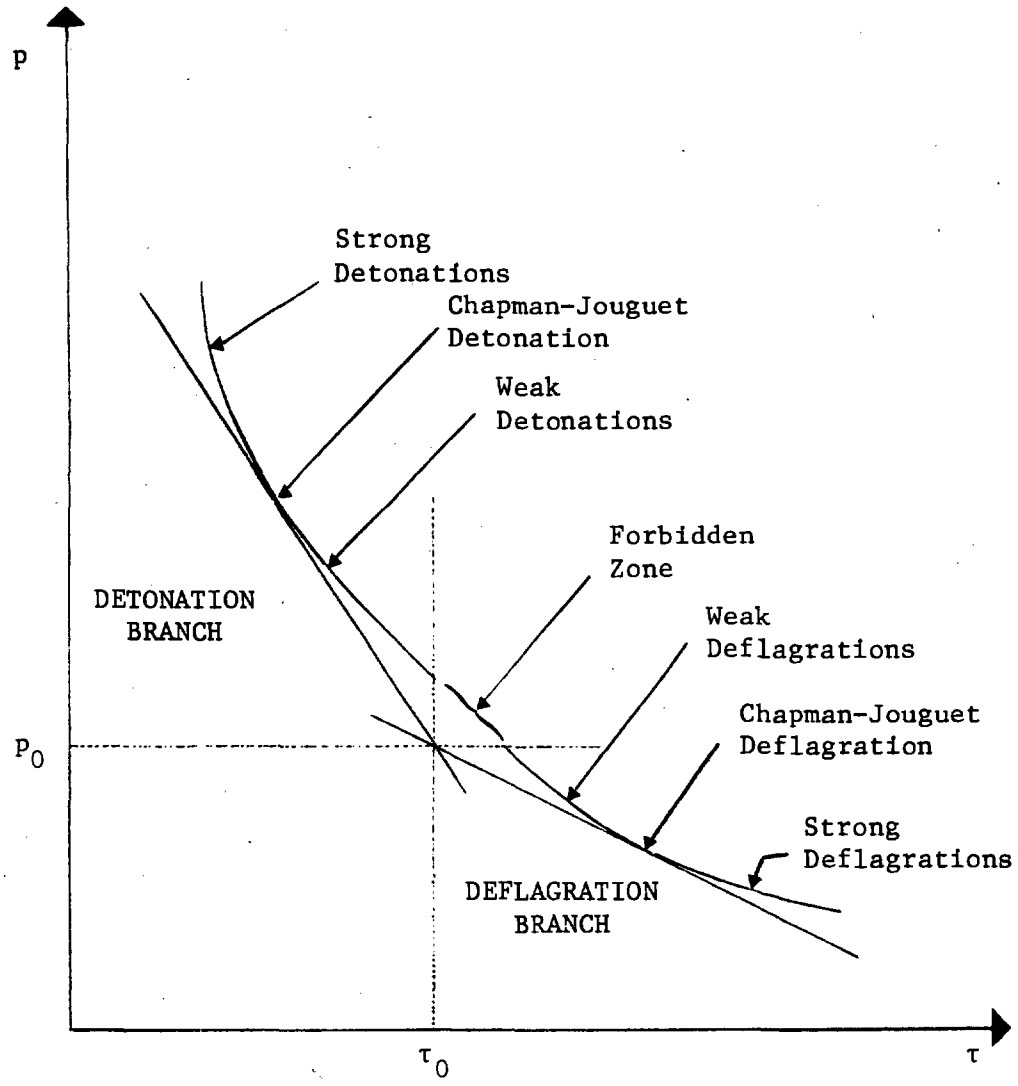

Figure A.1 Hugoniot curve for reacted gas

It should be noted that nothing has been said concerning the proximity of stations 0 and 1 or of the structure of the flow between them. It is assumed only that the flow is steady and that viscous stresses and heat conduction may be neglected at the two stations. In many cases of practical interest the flow may fail to be truly steady. Yet the present results may be regarded as approximately correct provided that the rates of change of mass, momentum and energy in the region bounded by stations 0 and 1 may be neglected by comparison with the fluxes of these quantities. Such conditions will be favored when the reaction zone is thin and the environment changes slowly.

From the conservation laws (A.1), (A.2) and (A.3) we may deduce two important relationships. First, from (A.1) and (A.2) we may deduce that

$$\frac{p_1 - p_0}{\tau_1 - \tau_0} = -m^2 \tag{A.4}$$

Thus it is apparent that only those processes are possible in which the sign of the change in pressure is opposed to that of the specific volume. Observe, moreover, that when the reaction zone has finite thickness, but is steady, and viscous stresses may be neglected, then (A.4) applies throughout the reaction zone and shows that the pressure and specific volume are linearly related throughout the process.

Next, introduce the Hugoniot function for the burnt gas

$$H^{(1)}(\tau,p) = E^{(1)}(\tau,p) - E^{1}(\tau_0,p_0) + \frac{1}{2}(\tau - \tau_0)(p + p_0)$$

Then (A.3) may be written as

$$H^{(1)}(\tau,p) = E^{(0)}(\tau_0,p_0) - E^{(1)}(\tau_0,p_0) \tag{A.5}$$

If the reaction is assumed to be exothermic as the gas flows from station 0 to station 1, it follows that the right hand side of (A.5) is positive.

Now suppose that the initial state τ_0,p_0 is given and regard (A.5) as a locus of possible final states. The graph of (A.5) appears as shown in figure A.1, subject to certain thermodynamic assumptions[8]. Because of (A.4) the accessible portion of (A.5) is confined to two separate branches, the upper corresponding to increases in pressure and the lower corresponding to decreases in pressure. These are referred to as the detonation and deflagration branches respectively. Equation (A.4) provides some immediate insight into an important difference between the two branches. It is apparent that the limiting constant pressure process --a deflagration--travels with an infinitesimal velocity. On the other hand, the limiting constant volume process--a detonation--travels with an infinite velocity. For the present discussion, only the deflagration branch is of interest.

The deflagration branch may itself be divided into two branches, identified in figure A.1 as weak and strong deflagrations, according to the magnitude of the pressure drop, and separated by a point identified as the Chapman-Jouguet deflagration. If one considers an arbitrary straight line emanating from (τ_0, p_0) and intersecting with the deflagration branch it follows, subject to certain thermodynamic assumptions [8], that the line has at most two points of intersection--one on the weak deflagration branch and the other on the strong deflagration branch. At the Chapman-Jouguet point, the straight line is tangent to the deflagration branch.

It may be shown that in the case of a weak deflagration, the reactants are subsonic relative to the reaction front and that in the case of a strong deflagration, they are supersonic. Naturally, the Chapman-Jouguet point is distinguished by the fact that the reactants are sonic with respect to the reaction front. In each case the speed of sound is understood to be that of the reacted gas. It is also shown by Courant and Friedrichs that in the case of an ideal gas, at least, a deflagration wave is always subsonic relative to the unreacted gas.

The foregoing discussion has distinguished between deflagrations with subsonic and supersonic reactants but has not identified any reasons to deny the admissibility of either. Indeed both the weak and strong deflagrations are clearly admitted by the principle of conservation of mass, momentum and energy. In order to show the impossibility of a steady strong deflagration wave it is necessary to consider the finite rate of reaction. Then the following geometrical argument may be employed.

Consider figure A.2. We show, schematically, the Hugoniot for the unreacted gas, the fully reacted gas, and the gas in several stages of partial reaction. That is to say, the preceding analysis is assumed to apply at all stages of the reaction. Now there are just two types of steady process which may occur between 0 and 1. The state may change continuously due to the finite rate of heat release by the reaction in which case we are required to traverse the straight line (A.4). Or, a compressive shock may occur, in which case we may traverse the Hugoniot corresponding to the state of reactedness at which the shock was encountered, the direction of the process being such as to lead to an increase in pressure.

It is evident, from the assumed shape of the Hugoniot curves illustrated in figure A.2, that a continuous process of heat release from 0 to 1 must inevitably intercept the weak deflagration branch when the reaction is complete. Accordingly, a transition to the second intersection point in the strong deflagration branch would require an expansion shock, which is inherently unstable. On the other hand, any admissible shock transition at a state of partial reactedness, can only lead to higher pressure from which only an intersection with the weak deflagration branch is possible, unless a detonation is admitted.

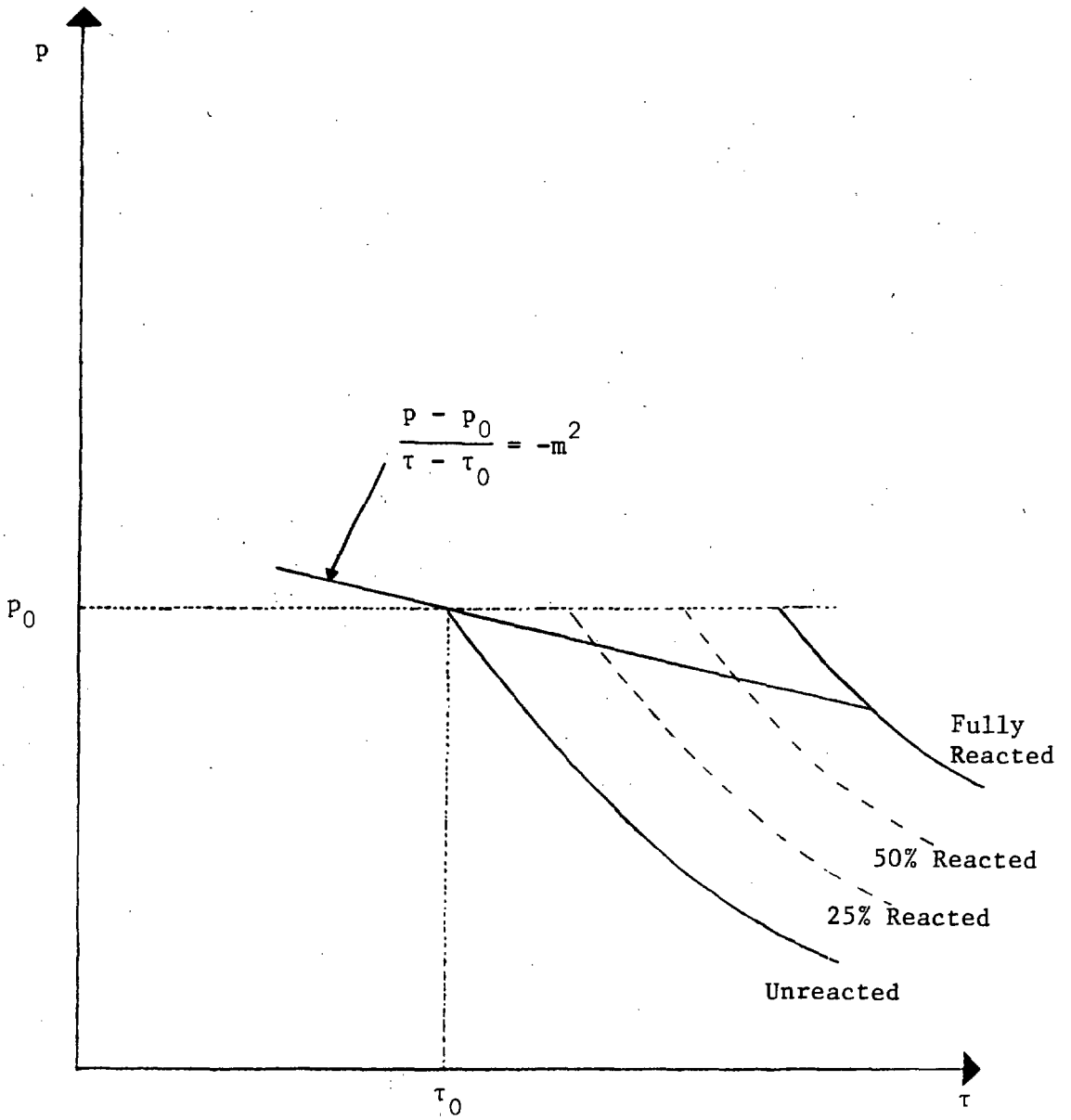

Figure A.2 Schematic demonstration of the implausibility of attaining a strong deflagration

This geometrical argument was also formalized by Courant and Friedrichs to provide an analytical demonstration that a simple flame model would reach a state of complete reactedness on the weak deflagration branch. The earlier work of Friedrichs[7] and the book by Williams[9] may also be consulted for demonstrations based on the structure of the solutions of particular models of the reaction zone. Landau and Lifschitz[10] have considered the problem from the point of view of the stability of the process. Considering the characteristic data on each side and assuming that just one internal condition is specified by the flame structure leads to the conclusion that a perturbation about a strong deflagration is underdetermined. Accordingly, exponentially growing solutions are admitted and the flame is unstable. The discussion of the inadmissibility of the strong deflagration wave given by Vinti[5] is essentially the same as the geometrical argument given here.

An alternative way of understanding the limitation of the reaction process to weak or at most sonic deflagrations follows from a consideration of the laws of choking in quasi-one-dimensional compressible flow. Moreover, this alternative point of view enables one to consider directly the consequences of a multiphase structure of the flame.

Consider a generalized quasi-one-dimensional steady flow in the manner described by Shapiro[18]. From the balance equations for a control volume as shown in figure A.3 one may deduce the following functional dependence of the Mach number M on the processes associated with the flow, namely cross sectional area, heat and mass addition and drag.

$$\frac{dM^2}{M^2} = -\frac{2\left[1 + \frac{\gamma - 1}{2} M^2\right]}{1 - M^2}\frac{dA}{A} + \frac{1 + \gamma M^2}{1 - M^2}\frac{dQ - dW_x + dH}{c_p T}$$

$$+ \frac{\gamma M^2\left[1 + \frac{\gamma - 1}{2} M^2\right]}{1 - M^2}\left\{4f\frac{dx}{D} + \frac{dX}{\frac{1}{2}\gamma p A M^2} - 2y\frac{d\omega}{\omega}\right\} \qquad (A.6)$$

$$+ \frac{2(1 + \gamma M^2)\left(1 + \frac{\gamma - 1}{2} M^2\right)}{1 - M^2}\frac{d\omega}{\omega} - \frac{1 + \gamma M^2}{1 - M^2}\frac{dW}{W} - \frac{d\gamma}{\gamma}$$

The nomenclature of this equation is taken directly from Shapiro and has the following significance.

M is the Mach number based on the ideal gas equation of state for which the speed of sound c is given by $c^2 = \gamma RT/W$.

A is the net flow area for the gas. That is, the area of the duct less the area of the entrained liquid or droplets or particles.

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[18] *Shapiro, A. "The Dynamics and Thermodynamics of Compressible Fluid Flow" Ronald Press, New York 1953*

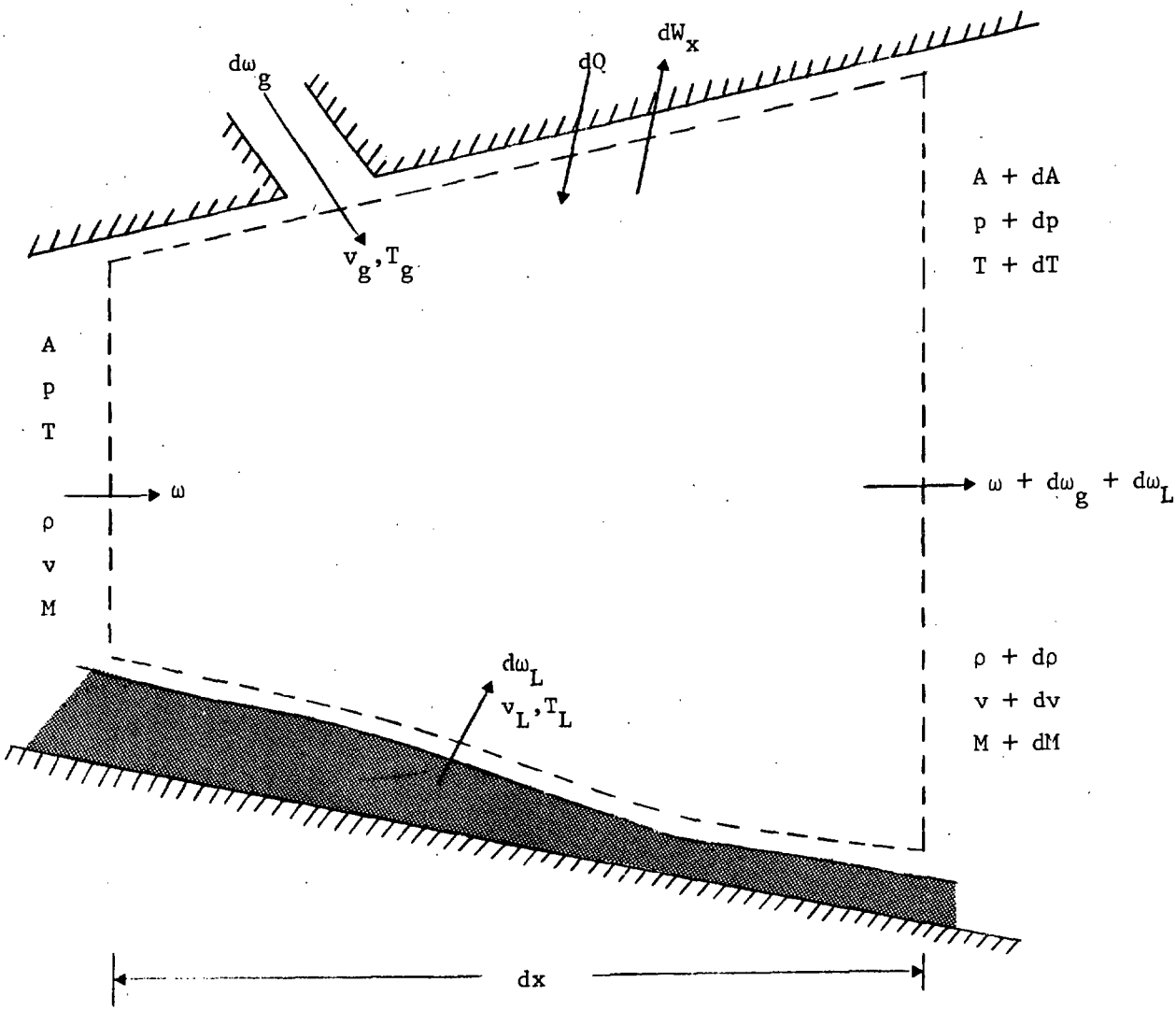

Figure A.3 Control volume for analysis of steady heterogeneous reacting flow (Shapiro, ref. 18)

γ is the ratio of specific heats

dQ is the net heat added to the gas stream by conduction or radiation from external sources, per unit mass of gas entering the control surface.

dW_x is the net external work to outside bodies per unit mass entering the control surface.

$$dH = dh_{pr} - [\bar{c}_{pg}\,(T - T_{og}) + v^2/2]\,\frac{d\omega_g}{\omega} - [h_L - h_v + \frac{v^2 - v_L^2}{2}]\,\frac{d\omega_L}{\omega}$$

dh_{pr} is the heat release per unit mass of gas stream due to decomposition, positive for an exothermic reaction.

f is the wall friction factor.

D is the hydraulic radius.

dX is the sum of the drag of stationary bodies, the drag of droplets, particles and filaments traveling more slowly than the gas stream, and the body or gravity forces.

$$y = [y_g\,\frac{d\omega_g}{\omega} + y_L\,\frac{d\omega_L}{\omega}]/(\frac{d\omega}{\omega})$$

$$y_g = v_g'/v \qquad y_L = v_L'/v$$

$d\omega = d\omega_L + d\omega_g$. See Figure A.3

W is the molecular weight.

R is the gas constant.

h_L is the enthalpy of liquid about to evaporate as it enters the control volume.

h_v is the enthalpy of evaporated liquid $d\omega_L$ at temperature T.

T_{og} is the stagnation temperature of the injected gas stream.

$\bar{c}_{pg}$ is the average value of c_p between T_{og} and T.

The remaining symbols may be identified from figure A.3 and the convention that v_g' and v_L' are the streamwise components of the velocities of the injected gas and liquid.

From a consideration of equation (A.6), which is seen to embed extremely complex behavior, the following conclusions may be drawn[18].

(1) An increase in area acts to decrease the value of M if $M < 1$ and to increase M if $M > 1$.

(2) Heat addition or combustion acts to increase M if $M < 1$ and to decrease M if $M > 1$.

(3) The effect of friction, or drag of internal bodies, acts to increase M if $M < 1$ and to decrease M if $M > 1$.

(4) Mass addition with $y < 1$ acts to increase M if $M < 1$ and to decrease M if $M > 1$.

(5) An increase in W acts to decrease M if $M < 1$ and to increase M if $M > 1$.

(6) An increase in γ always acts to reduce M.

These results permit us to understand the denial of the admissibility of the strong deflagration wave from an alternative point of view. The previous discussion, based on the assumed shape of the Hugoniot of the reactants, corresponds to a process in which only heat addition due to combustion was of interest. Evidently, if we consider an arbitrary initial state, presumed subsonic, the Mach number increases steadily as we pass towards the fully reacted state. If, however, the Mach number increases to unity, there is no longer a solution to the steady flow problem unless the initial state may be altered. Thus the inadmissibility of the strong deflagration wave is perceived to be a manifestation of the well-known phenomenon of choking.

In the heterogeneous flame we must also consider mass addition and drag. These, too, always drive the Mach number towards unity just as the heat addition does. Only one possibility appears to exist in the heterogeneous flame whereby the sonic point can be passed in a continuous manner. The cross-sectional area of the flow will increase due to consumption of the dispersed condensed phase and also, possibly, due to separation of the droplets or particles as a consequence of the drag. If, as the sonic point is encountered, the effect of change of area is such as to dominate the opposing effects of heat addition, mass addition and drag, then a continuous transition to supersonic flow would be possible. A trivial example of such a case is, of course, the converging-diverging rocket nozzle in which the cross-sectional area of the duct is used to effect the transition from subsonic to supersonic flow.

The possibility of obtaining such continuous transitions by means of the area change associated with condensed phase fuel consumption and dispersal may be determined only by parametric studies of equation (A.6). Such studies are beyond the scope of the present enquiry. However, we may speculate that the conditions under which the transition could be made would correspond to fuels of rather low ballistic efficiency in which inert components would play the role of a conventional rocket nozzle.

We conclude with some comments on the implications of these results as regards the end-burning traveling charge. First, we concur with Vinti's conclusion that the Langweiler process would be inherently incapable of realization once the projectile velocity exceeded the speed of sound in the reacted gas. Only by making the reaction zone sufficiently thick that unsteady effects become important can we expect to develop supersonic reactants. Of course, as the reaction zone becomes

thicker, we approach the configuration of a conventional propelling charge in which, as discussed in the introduction to this report, the reaction zone effectively fills the tube. However, the rejection of the Langweiler scheme does not necessarily defeat the concept of the traveling charge as a potentially useful ballistic solution. Because the thrust associated with the Langweiler scheme increases continually as the projectile accelerates, the gun is required to operate at a condition wich is far removed from the ideal constant pressure cycle. Moreover, the pressure on the unreacted side of the gas/propellant interface is found to rise very sharply as the Mach number increases beyond unity.

Hence, the present findings are more appropriately considered in the context of an ideal constant base pressure scheme for the burning rate. In this context, the denial of the possibility of a strong deflagration wave is not necessarily restrictive. It is merely necessary that the Chapman-Jouguet point be reached so that the combustion zone is just uncoupled from the pressure of the column of reactants. In such a case the propellant will burn under a choked condition. However, it seems probable that the pressure of the unreacted propellant would be influenced not only by the chemical formulation of the propellant but also by its mechanical behavior during the interior ballistic cycle.

Appendix B: Code Description and Fortran Listing

The model of the end burning traveling charge has been encoded in the FORTRAN IV language for implementation on the CYBER 7600 computer and is documented by the listing which forms the principal part of this appendix. We provide a summary of the routines, their purpose and their linkages to other routines, in Table B.1. In Table B.2 we provide a glossary of those variables which are contained in the common block areas of storage. Local variables are not described. Table B.3 summarizes, in detail, the input files used to run the program.

In addition to the tabular information, we provide the following brief discussion of the code structure, paying particular attention to the manner in which the physical problem is represented. We also comment on the code output.

The program consists of a main routine, TCMAIN, which is supported by a total of twenty subfunctions and subprograms. For ease of maintenance, particularly in connection with the constitutive laws, a largely modular approach has been followed in writing the code. Only the covolume equation of state of the gas is intrinsically bound into the code; the remaining constitutive laws are expressed by individual subroutines. With the exception of subroutine BASE, which embeds the rather complicated logic associated with the various combustion models and the branching among them, the programming is extremely straightforward.

The principal computational arrays are GS1(100,3), GS2(100,3) and GS3(100,3) as described in Table B.2. The first index of these arrays points to a mesh location, a maximum of 100 points being admitted by the present dimension statements. The second index points to a storage level and is assigned the values 1,2,3 on a rotating basis. At any level, the index NI points to current storage; that is to say, NI points to the present data at the outset of a predictor step and is advanced to point to the predicted future data on the corrector step. The index NF points to the storage level which contains the future data. Finally, the index NP is used, on the corrector step, to point to the present data. The principal integration step counter is NDT, which is incremented by one on each predictor and each corrector level.

At any time, the gas column is represented by NDIM mesh points whose data occupy the first NDIM locations of GS1, GS2 and GS3. If the propellant is treated as a continuum, its data occupy the storage locations NDIM1 to NDIM2 of GS1, GS2 and GS3, where NDIM1 = NDIM + 1.

The execution of the program is controlled by the data described in Table B.3. Multiple runs may be performed as follows. For a given problem, a parametric series may be run using an array of values of the ratio of charge-to-projectile mass. At the conclusion of this series, the program returns to its starting point and looks for another complete set of data. If none is furnished, exit occurs. When a parametric series is to be conducted, the input datum IDEALI, which defines the

combustion model, is also consulted. The basic values of IDEALI are zero, if measured burn rate data are used; one, if Langweiler burning is used; and two, if the pressure on the unreacted side of the gas/propellant interface, or the projectile acceleration is specified. By setting IDEALI equal to three or four, initial conditions are deduced, using the relations described in section 2.6, to yield the appropriate value of pressure or acceleration in combination with the designated charge-to-projectile mass ratio. Following the evaluation of the initial data in such cases, IDEALI defaults internally to the value two. It should also be noted that when the initial properties are deduced from the charge-to-projectile mass ratio, the propellant density is determined from a user-supplied value of the loading density and may differ from the value typical of the homogeneous substance.

We also note, in regard to those data which are defined in a tabular fashion, that, except for the measured burn rate data, the following interpretation is made in a consistent manner. Values of the dependent variable are deduced for an argument lying within the table range of the independent variable by means of linear interpolation. When an argument is furnished which lies below or above the table range, the dependent variable is assigned the first or the last entry, accordingly. The measured burn rate coefficients are assumed to be constant until the mass fraction consumed at any time exceeds the corresponding value ZR assigned to the coefficients. Interpolation does not occur.

We conclude by commenting on the output of the code. The principal logical index in this regard is the input quantity NPRO. If it is zero, the continuum state variables are printed at times which are integer multiples of the input quantity TINOM. By setting NPRO equal to one, two or three, various bodies of compressed output can be obtained in place of the full tables of continuum data. If NPRO is four, no intermediate results are printed. In all cases, the calculation concludes with a table of summary data of ballistic interest. The code also tracks the total mass and energy of the system at any time. At the conclusion of the run the maximum and final percentage defects of these quantities, from the initial values, are printed. These may be used to assess the approximate magnitude of the errors of numerical integration in cases where physical losses do not occur due to dissipation or outflow.

Table B.1 Summary of Routines - Purpose and Linkages

TCMAIN

Called by: NONE
Calls: BAR1, BAR2, BAR3, BAR4, BASE BASEPR, BREECH, GETK, OUTFLO, REFIT, SIGCHK, VELCHK
Purpose: Main program and principal executive routine. TCMAIN reads and prints problem data, initializes constants, and then either calls BAR1 to initialize the state variables or reads data from unit 8 if problem is restarted. TCMAIN organizes the two-level integration scheme and is supported by specialized subroutines which handle interior mesh points (BAR3) and the various boundary conditions (BASE, BASEPR, BREECH, OUTFLO). TCMAIN provides printed output and disc storage of solution in accordance with user-supplied data. Summary data are accumulated and are printed at the termination of each case.

BAR1

Called by: TCMAIN
Calls: BURN, RNOM, REFIT, RESP
Purpose: BAR1 initializes continuum arrays and, through the call to REFIT, establishes the initial mesh.

BAR2

Called by: TCMAIN, BAR3, CARAC
Calls: None
Purpose: Utility routine, BAR2 transforms computational variables into ordinary state variables.

BAR3

Called by: TCMAIN
Calls: BAR2, HTW, WFR
Purpose: Integration of all continuum equations at all interior mesh points. Called twice per integration step, once for each level.

BAR4

Called by: TCMAIN
Calls: DSDR
Purpose: Computes maximum time step allowable according to Courant-Friedrichs-Lewy stability condition and safety factor SAFE.

BASE

Called by: TCMAIN
Calls: BURN, CARAC, DSDR, RESP, RNOM
Purpose: Subroutine BASE is responsible for the determination of the boundary values at the base of the projectile, following burnout, and at the gas/propellant interface for all the combustion models. If the propellant is treated as a continuum, its boundary values at the interface are also deduced by BASE. The routine is called twice per update step, once at each level.

BASEPR

Called by: TCMAIN
Calls: CARAC, DSDR, RNOM
Purpose: Updates boundary values at base of projectile when the propellant is treated as a continuum. Called twice per integration step.

BREECH — Called by: TCMAIN
Calls: CARAC
Purpose: Updates boundary values at breech of tube when breech is closed or impermeable to gas.

BURN — Called by: BAR1, BASE
Calls: None
Purpose: Computes measured burn rate as a function of pressure of reactants.

CARAC — Called by: BASE, BASEPR, BREECH, OUTFLO
Calls: BAR2, DSDR
Purpose: Given a value of the velocity at a boundary mesh point, CARAC deduces a value of pressure which is compatible with the given value and also with the flow in the interior.

DSDR — Called by: BAR4, BASE, BASEPR, CARAC
Calls: SNOM
Purpose: Computes wave speed in propellant as a function of density and rate-of-change of density.

GETK — Called by: TCMAIN
Calls: RESP
Purpose: Computes time derivatives of state variables governed by ordinary differential equations. Called twice per integration step, once at each level.

HTW — Called by: BAR3
Calls: None
Purpose: Computes heat loss per unit volume due to heat transfer from gas to tube wall.

OUTFLO — Called by: TCMAIN
Calls: CARAC
Purpose: Computes boundary values at gas permeable breech and also at muzzle following expulsion of the projectile. Called twice per integration step.

REFIT — Called by: TCMAIN, BAR1
Calls: None
Purpose: Allocates mesh and performs spline interpolation of data when mesh changes.

RESP — Called by: BAR1, BASE, GETK
Calls: None
Purpose: Computes resistance to projectile motion due to friction on obturator and pressure of shocked air in barrel.

RNOM — Called by: BAR1, BASE, BASEPR
Calls: None
Purpose: Computes propellant density as a function of pressure on nominal loading curve.

SIGCHK — Called by: TCMAIN
Calls: SNOM
Purpose: Checks that state of continuum propellant does not lie above nominal loading curve.

SNOM — Called by: DSDR, SIGCHK
Calls: None
Purpose: Computes propellant pressure as a function of density on nominal loading curve.

VELCHK — Called by: TCMAIN
Calls: None
Purpose: Checks that continuum propellant velocity has not reversed sign as a consequence of friction.

WFR — Called by: BAR3
Calls: None
Purpose: Computes friction between continuum propellant and tube wall.

Table B.2 Glossary of Principal Fortran Variables

AB	Bore Area
ADWN	Unload/Reload wave speed in propellant
AIRGAM	Ratio of specific heats of air in barrel in front of projectile
AIRMW	Molecular weight of air in barrel
AIRP0	Pressure of unshocked air in barrel
AIRT0	Temperature of unshocked air in barrel
AMU(10)	Array of coefficients of friction between propellant and tube
AMUV(10)	Array of velocities corresponding to AMU
ANU	Poisson ratio of projectile
APMAX	Maximum allowable acceleration of projectile
ASBR	Throat area of gas-permeable breech
AUP	Loading wave speed in propellant
BMU(10)	Array of coefficients of friction between obturator and tube wall
BMUV(10)	Array of velocities corresponding to BMU
BR(10)	Array of resistive pressures due to obturator
BRX(10)	Array of projectile displacements corresponding to BR
BN(20)	Array of burn rate exponents
BV	Covolume of gas
B1(20)	Array of burn rate additive constants
B2(20)	Array of burn rate pre-exponential factors
CDBR	Discharge coefficient of gas-permeable breech
CDMUZ	Discharge coefficient of muzzle

CHTW	Coefficient of wall heat transfer correlation
CM	Unreacted propellant mass
CV	Specific heat of gas at constant volume
DB	Diameter of barrel
DELTAX	Non-dimensional mesh spacing in gas column
DELTBX	Non-dimensional mesh spacing in propellant
DELYR	Thickness of gas film used to lubricate propellant
DRHO(2)	Total time derivative of density of propellant at boundaries
DXMIN	Minimum allowable mesh spacing in physical plane
E(100)	Array of values of internal energy
ECHEM	Chemical energy of propellant
ELB	Length of bearing section of obturator
ETA(100)	Array of values of velocity of mesh
GAM	Ratio of specific heats of gas
GS1(100,3)	Computational array. If I corresponds to the gas GS1(I,J) contains the quantity ρx_b at the I-th mesh point at the J-th level of integration. Otherwise GS1(I,J) contains the value of ρ_p
GS2(100,3)	Computational array. If I corresponds to the gas, GS2(I,J) contains the quantity $\rho u x_b$ at the I-th mesh point and the J-th integration level. Otherwise GS2(I,J) contains u_p.
GS3(100,3)	Computational array. If I corresponds to the gas, GS3(I,J) contains the quantity $\rho x_b(e + u^2/2g_o)$ at the I-th mesh point and the J-th level of integration. Otherwise, GS3(I,J) contains σ.
IDEAL	Burn rate indicator. See File 2 discussion of IDEALI.
INT	Set equal to 1 on predictor level and 0 on corrector level of integration step

K(4,2)	Array of derivatives of X_p, $\dot{X}_p$, M_p and x_p at predictor and corrector levels.
MACH	Maximum allowable Mach number of reactants relative to regressing interface
MAXD1M	Maximum allowable number of mesh points
MOL	Molecular weight of gas
NBR	Number of increments of propellant for tabular description of burn rate coefficients
NBRES1	Number of entries in tabular description of resistance due to obturator
NBRES2	Indicator for calculation of resistance due to shocked air in front of projectile
NBRES3	Number of entries in tabular description of coefficient of friction of obturator as a function of projectile velocity
NBRV	Indicator that breech is gas-permeable or otherwise
NCJ	Error indicator
NDIM	Number of mesh points allocated to gas
NDIM1	NDIM + 1
NDIM2	Total number of mesh points allocated when propellant is treated as a continuum
NDT	Integration counter. Incremented on both predictor and corrector levels
NF	Pointer to next integration level of storage
NHTW	Indicator that wall heat loss is considered or otherwise
NI	Pointer to current integration level storage
NMUZBL	Indicator that blowdown is to be computed or otherwise
NP	Pointer to previous integration level storage

NPRC	Indicator that propellant is treated as continuum or otherwise
NPRO	Print option indicator. See File 2.
NWFR	Indicator of representation of friction between solid propellant and tube. If positive, number of entries in tabular description of coefficient of friction as a function of velocity of propellant
P(100)	Array of values of pressure
PBRF	Value of breech pressure at which blowdown computation is terminated
PDIA	Value of pressure which must be exceeded in breech prior to onset of permeability
PRM	Projectile mass
PRMB	Projectile mass ahead of midpoint of obturator
PST	Initial pressure of gas
QORF(100)	Array of values of heat loss, if mesh point corresponds to gas, and wall friction, if point corresponds to propellant
RDOT	Rate of regression of interface relative to unreacted propellant
RHO(100)	Array of values of density
RHOP	Initial density of propellant at zero pressure
SIG	Value of pressure on unreacted side of gas/propellant interface
SIGMAX	Maximum allowable value of SIG
SSTART	Shot start pressure
T(100)	Array of values of temperature
U(100)	Array of values of velocity
XB	Length of gas column
XBB	Length of propellant column

XI1	$\frac{\gamma}{\gamma - 1} \frac{p_o}{\rho_p}$
XI2	$e_p - \frac{p_o}{\gamma - 1} (\frac{1}{\rho_p} - b)$
XLPRI	Initial length of propellant column
XPR	Projectile travel
XPROP	Initial position of base of projectile relative to breech
ZR(20)	Array of values of mass fraction of propellant for tabular description of burn rate coefficients

Table B.3 Description of Input Files

File 1: One Card (20A4) Mandatory	
ITIT	Problem title. Up to 80 alphanumeric characters

File 2: One Card (16I5) Mandatory	
IDEALI	Propellant burn rate indicator 0 - Measured burn rate data. File 6 required. 1 - Langweiler ideal burning 2 - Ideal burning with prespecified value of pressure on unreacted side of gas/propellant interface or of projectile acceleration. Note the discussion of SIGMAX, MACH, APMAX in File 4. 3 - Like 2 except that APMAX is deduced from SIGMAX according to initial propellant mass. Option used for parametric studies in which SIGMAX is to be constant while charge-to-projectile mass ratio varies. 4 - Like 2 except that PSTI (File 4) and SIGMAX are computed from APMAX according to initial propellant mass. Option used for parametric studies in which APMAX is to be held constant while charge-to-projectile mass ratio varies.
NPRC	0 - Propellant treated as rigid 1 - Propellant treated as continuum. File 7 required.
NPRO	Print Option for logout other than summary data 0 - Detailed print including flow profiles 1 - One line summary at each logout step 2 - Energy trajectory printed 3 - Interior ballistic data and % energy trajectory printed 4 - No print other than summary
NDSK	Disc read/write parameter. If active, unit 8 must be defined. 0 - Neither read nor write 1 - Write only 2 - Read only 3 - Read and write
NDSKID	Problem identifier if multiple storage on unit 8
NDSKDT	Time step for restart of problem if NDSK = 2 or 3
NPAR	Number of parametric cases. If NPAR > 0, File 17 is required.
NBR	Number of entries in burning rate table, File 6. Default value is NBR = 1 (Maximum of 20)

NWFR	Propellant Wall Friction Parameter 0 - Friction between propellant and tube not considered -1 - Friction due to gas film. File 8 required. >0 - Number of entries in velocity dependent coefficient of friction table. (Maximum of 10). File 9 required.
NBRES1	0 - Obturator resistance not given as table >0 - Number of entries in table of resistive pressure versus travel (Maximum of 10). File 10 required.
NBRES2	0 - Resistance due to shocked air not considered 1 - Resistance due to shocked air considered. File 11 required.
NBRES3	0 - Obturator resistance not proportional to setback pressure. >0 - Number of entries in table of velocity dependent coefficient of friction of obturator. (Maximum of 10). Files 12 and 13 required.
NHTW	0 - Heat loss to wall not considered. 1 - Heat loss considered. File 14 required.
NMUZBL	0 - Tube blowdown after muzzle exit not considered. 1 - Blowdown considered. File 15 required.
NBRV	0 - Breech closed. 1 - Breech gas-permeable. File 16 required.

File 3: One Card (4I5, 5F10.0) Mandatory

NSTOP	Number of integration steps before termination. If NSTOP = 99999, number is unbounded.
MAXDIM	Maximum number of mesh points to be used in continuum representations. (≤ 100)
NCY1	Following step NCY1, NPRO will default internally to 0 to yield detailed printing.
NCY2	Following step NCY2, NPRO resumes the value specified in File 2.
TINOM	Time interval for printing (See NPRO, File 2) and disc storage (See NDSK, File 2). (msec)
TISTOP	Time at which computation is to be terminated (msec)
XSTOP	Projectile travel at which computation is to be terminated (ins)

SAFE	Safety factor by which C-F-L time step is divided. Must be at least one.
DXMIN	Minimum mesh size for continuum representation (ins).

File 4: One Card 8F10.0)	Mandatory
DB	Diameter of tube (ins)
XIB	Initial length of gas column (ins)
PRM	Mass of projectile
SSI	Shot start pressure (psi)
PSTI	Initial pressure of gas (psi)
SIGMAX	Maximum value of pressure on unreacted side of gas/propellant interface. If SIGMAX = 0, no restriction is considered. (psi)
MACH	Maximum value of Mach number of reactants relative to regressing surface. If MACH = 0, no restriction is considered.
APMAX	Maximum value of acceleration of projectile. If APMAX = 0, no restriction is considered. (gravities)

File 5: One Card (8F10.0)	Mandatory
GAM	Ratio of specific heats of gas (-)
BV	Covolume (in^3/lbm)
MOL	Molecular weight (lbm/lbmol)
ECHEM	Chemical energy of propellant (lbf-in/lbm)
RHOP	Density of solid propellant at zero pressure (lbm/in^3)
CM	Mass of propellant (lbm)
ALDEN	Loading density (lbm/in^3)

File 6: NBR Cards (4F10.0)	Required if and only if IDEALI = 0 or if NBR is read as a positive number. See File 2.

B1(1)	Burn rate additive constant for 1st increment of propellant (in/sec)
B2(1)	Burn rate pre-exponential coefficient for 1st increment of propellant (in/sec - psi^{BN})
BN(1)	Burn rate exponent for 1st increment of propellant (-)
ZR(1)	Mass fraction defined by end of first increment (-)
B1(2)	Like B1(1) but for 2nd increment. (New Card)
.	
.	
.	
ZR(NBR)	Mass fraction defined by end of last increment, including contributions of all preceding increments (-)

File 7: One Card (2F10.0) Required if and only if NPRC ≠ 0. See File 2.

AUP	Compressive wave speed in propellant at ambient conditions (in/sec)
ADWN	Unloading/Reloading wave speed (in/sec). If ADWN is entered so that it is less than the nominal loading wave speed, the loading value is used. By entering ADWN = 0 a reversible law is defined.

File 8: One Card (2F10.0) Required if and only if NWFR < 0. See File 2.

VISLYR	Viscosity of gas film used to lubricate propellant (lbm/in-sec)
DELYR	Thickness of film (ins)

File 9: One to three Cards (8F10.0) Required if and only if NWFR > 0. See File 2.

AMUV(1)	First value of velocity of propellant (in/sec)
AMU(1)	Corresponding coefficient of friction on tube (-)
.	
.	
.	
AMUV(NWFR)	Last value of velocity (in/sec)
AMU(NWFR)	Corresponding coefficient of friction (-)

File 10: One to three Cards (8F10.0)	Required if and only if NBRES1 ≠ 0. See File 2.
BRX(1)	First value of projectile travel (ins)
BR(1) . . .	Corresponding value of resistive pressure due to obturator (psi)
BRX(NBRES1)	Last value of projectile travel (ins)
BR(NBRES1)	Corresponding value of resistive pressure (psi)

File 11: One Card (4F10.0)	Required if and only if NBRES2 ≠ 0. See File 2.
AIRGAM	Ratio of specific heats of air (-)
AIRPO	Pressure of air in barrel (psi)
AIRTO	Temperature of air in barrel (°R)
AIRMW	Molecular weight of air in barrel (lbm/lbmol)

File 12: One Card (3F10.0)	Required if and only if NBRES3 ≠ 0. See File 2.
PRMB	Mass of projectile ahead of midpoint of obturating band (lbm)
ELB	Length of bearing section of obturating band (ins)
ANU	Poisson's ratio of obturating band (-)

File 13: One to three Cards (8F10.0)	Required if and only if NBRES3 ≠ 0. See File 2.
BMUV(1)	First value of velocity of projectile (in/sec)
BMU(1) . . .	Corresponding value of coefficient of friction between obturator and tube (-)
BMUV(NBRES3)	Last value of velocity of projectile (in/sec)
BMU(NBRES3)	Corresponding coefficient of friction (-)

File 14: One Card (4F10.0)	Required if and only if NHTW ≠ 0. See File 2.

TWALO	Temperature of tube (°R)
CHTW	Coefficient of heat transfer correlation (-) Default value is 0.092.
VISG	Viscosity of gas (lbm/in-sec) Default value is 10^{-5}.
PRNO	Prandtl number of gas (-) Default value is 0.7.

File 15: One Card (2F10.0)	Required if and only if NMUZBL ≠ 0. See File 2.

PBRF	Value of breech pressure at which blowdown calculation is to be terminated (psi)
CDMUZ	Discharge coefficient for efflux from muzzle (-)

File 16: One Card (4F10.0)	Required if and only if NBRV ≠ 0. See File 2.

ASBR	Throat area of discharge nozzle in breech (in^2)
CDBR	Discharge coefficient for nozzle (-)
PDIA	Rupture pressure which must be exceeded before breech becomes permeable to gas (psi)

File 17: One or two Cards (8F10.0)	Required if and only if NPAR ≠ 0. See File 2.

COM(1)	Value of charge-to-projectile mass ratio for first case (-).
.	
.	
.	
COM(NPAR)	Value for last case (-)

```
C      PROGRAM TO STUDY END BURNING TRAVELING CHARGE
C
C      VERSION CREATED FEB 1 1980.
C
       IMPLICIT REAL*8(A-H,O-Z)
       REAL*8 MOL, K, MACH
C
       COMMON /BARA/ GS1(100,3), GS2(100,3), GS3(100,3)
       COMMON /BAR3/ RHO(100), P(100), E(100), T(100), U(100), ETA(100)
       COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
       COMMON /BARD/ DXMIN, DELTAX
       COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1   CV, ZR(20)
       COMMON /BARF/ XPR, VPR, CM, XB, VB, AB, PRM, RDOT, SIG, SSTART
       COMMON /BARG/ XBR,DELTBX,NPRC,NDIM1,NDIM2
       COMMON /BARH/ AUP,ADWN,VISLYR,DELYR,AMUV(10),AMU(10),DB,
     *              BRX(10),BR(10),AIRGAM,AIRPO,AIRTO,AIRMW,TWALO,
     *              PBRF,CDMUZ,ASBR,CDBR,PDIA,CHTW,DRHO(2)
       COMMON /BARH2/ PRMB,ELB,ANU,BMUV(10),BMU(10),NBRES3
       COMMON /BARH3/ PRND,VISG
       COMMON /BARH4/ RESAIR,RESOB
       COMMON /BARI/ NWER,NBRES1,NPRES2,NHTW,NMUZBL,NBRV
       COMMON /BARCJ2/ PST, SIGMAX, MACH, APMAX, XI1, XI2, NCJ, IDEAL
       COMMON /BARJ/ QORF(100),PRPRES
       COMMON /BERG/ K(4,2)
       COMMON /LPRO/ NPRO
       COMMON /BRVAR/ XLPRI, XPROP,NBR
C
       DIMENSION DT1(5), SAVE(5), PDS(100),NBC(2)
       DIMENSION COM(10), ITIT(20)
       DIMENSION SEGW(100), SEGE(100), SEGKE(100)
C
       EQUIVALENCE (DT1(1),XPR)
       DATA R, G /19550.D0, 386.16D0/
C
C      READ TITLE LINE
C
   20  READ (5,450,END=410) (ITIT(I),I=1,20)
C
C      READ AND PRINT INPUT DATA
C
       READ(5,490) IDEALI,NPRC,NPRO,NDSK,NDSKID,NDSKDT,NPAR,NBR,
     *             NWER,NBRES1,NBRES2,NBRES3,NHTW,NMUZBL,NBRV
       READ(5,495) NSTOP,MAXDIM,NCY1,NCY2,TINOM,TISTOP,
     *             XSTOP,SAFE,DXMIN
       NPROI=NPRO
       IDEAL=IDEALI
C
C      READ IN SINGLE SET OF VALUES IF C/M DOES NOT VARY.
```

```
      READ (5,500) DB,XIB,PRM,SSI,PSTI,SIGMAX,MACH,APMAX
      READ (5,500) GAM,BV,MDL,ECHEM,RHOP,CM,ALDEN
      IF(IDEALI.NE.0) GO TO 40
      NBRA=1
      IF(NBR.GT.0) NBRA=NBR
      DO 30 I=1,NBRA
      READ(5,500)B1(I),B2(I),BN(I),ZR(I)
   30 CONTINUE
   40 IF(NPRC.NE.0) READ(5,500) AUP,ADWN
      IF(NWFR.LT.0) READ(5,500) VISLYR,DELYR
      IF(NWFR.GT.0) READ(5,500) (AMUV(I),AMU(I),I=1,NWFR)
      IF(NBRES1.NE.0) READ(5,500) (BRX(I),BR(I),I=1,NBRES1)
      IF(NBRES2.NE.0) READ(5,500) AIRGAM,AIRPO,AIRTO,AIRMW
      IF(NBRES3.EQ.0) GO TO 45
      READ(5,500) BRMB,ELB,ANU
      READ(5,500) (BMUV(I),BMU(I),I=1,NBRES3)
   45 IF(NHTW.NE.0) READ(5,500) TWALO,CHTW,VISG,PRNO
      IF(VISG.LT.1.D-10) VISG=1.D-5
      IF(PRNO.LT.1.D-10) PRNO=0.700
      IF(CHTW.LT.1.D-10) CHTW=0.092D0
      IF(NMUZBL.NE.0) READ(5,500) PBRF,CDMUZ
      IF(NBRV.NE.0) READ(5,500) ASBR,CDBR,PDIA
      NPR=1
      FOR=ECHEM*(GAM-1.D0)
      AB=3.1415926D0/4.D0*DB*DB
      VI=AB*XIB
      CMI=CM
      CT=CM+VI*PSTI/(FOR+PSTI*BV)
      IF (NPAR.EQ.0) GO TO 80
      READ(5,500)(COM(I),I=1,NPAR)
   60 CT=COM(NPR)*PRM
C
CCC   COMPUTE SIGMAX AND PSTI BASED ON APMAX
C
      IF (IDEALI.EQ.4) IDEAL=2
      IF (IDEALI.NE.4) GO TO 70
      CPR=APMAX*(PRM+CT)/AB
      TP1=VI*APMAX/AB+FOR-CPR*BV
      TPSQ=TP1*TP1
      TP2=4.D0*BV*CPR*FOR
      PSTI=(-TP1+DSQRT(TPSQ+TP2))/(2.D0*BV)
      SIGMAX=PSTI
   70 CI=PSTI*VI/(FOR+PSTI*BV)
      VC=CT/ALDEN
      CM=CT-CI
      RHOP=CM/(VC-VI)
      CMI=CM
   80 PST=PSTI
      CTOT=CT
      PTOT=CT*ECHEM
```

```
   90 WRITE (6,460) (ITIT(I),I=1,20)
      WRITE(6,520) IDEALI,NPRC,NPRO,NDSK,NDSKID,NDSKDT,NPAR,NBR,
     *             NWFR,NPRES1,NBRES2,NBRES3,NHTW,NMUZBL,NBRV
      WRITE(6,525) NSTOP,MAXDIM,NCY1,NCY2,TINOM,TISTOP,
     *             XSTOP,SAFE,DXMIN
C
CC    COMPUTE APMAX BASED ON SIGMAX
C
      IF (IDEALI.EQ.3) IDEAL=2
      IF (IDEALI.EQ.3) APMAX=SIGMAX*AB/(PRM+CM)
      WRITE (6,530) DB,XIB,PRM,CM,PST,SIGMAX,MACH,APMAX,COM(NPR),VC,
     1  ALDEN,SSI
      WRITE(6,460) (ITIT(I),I=1,20)
C
CC    INITIALIZE CONSTANTS
C
      TNOM=1.D-3*TINOM
      TSTOP=1.D-3*TISTOP
      LINE=0
      NSTEP=0
      SSTART=SSI
      NPRT=0
      NEXIT=0
      DUM=0.D0
      PMAX=PST
      PMAXA=PST
      NBRT=0
      XB=XIB
      NBC(1)=0
      NBC(2)=0
      IF (CM.LT.1.D-10) NPRT=2
      IF (MOL.LT.1.D-10) MOL=28.D0
      CV=R/MOL/(GAM-1.D0)
      TIME=0.D0
      TPRT=0.D0
      EDPT=0.D0
      SIG=0.D0
      VB=0.D0
      XPR=0.D0
      VPR=0.D0
      QLOSS=0.D0
      RESLS1=0.D0
      RESLS2=0.D0
      PRPRES=0.D0
      STMAX=0.D0
      AKGMAX=0.D0
      RPMAX=0.D0
      NNPRC=NPRC
      NNBC2=0
      RESAIR=0.D0
```

```
      RESOB=0.DO
      DPMAX=0.DO
      DPMIN=0.DO
      CTDELM=0.DO
      ETDELM=0.DO
      DRHO(1)=0.DO
      DRHO(2)=0.DO
      DO 94 I=1,8
   94 K(I,1)=0.DO
      DO 95 I=1,100
   95 QORF(I)=0.DO
      XI1=GAM/(GAM-1.DO)*PST/RHOP
      XI2=ECHEM-PST/(GAM-1.DO)*(1.DO/RHOP-BV)
      XLPRI=CM/(RHOP*AB)
      XBB=XLPRI
      XPROP=CM/RHOP/AB+XB
      IF(NBRES2.NE.0) WRITE(6,840) AIRGAM,AIRPO,AIRTO,AIRMW
      IF(NHTW.NE.0) WRITE(6,850) TWALO,CHTW,VISG,PRNO
      IF(NMUZBL.NE.0) WRITE(6,860) PBRF,CDMUZ
      IF(NBRV.NE.0) WRITE(6,870) ASBR,CDBR,PDIA
      IF(NBRES1.NE.0) WRITE(6,830) (BRX(I),BR(I),I=1,NBRES1)
      IF(NBRES3.NE.0) WRITE(6,845) PRMB,ELB,ANU,(BMUV(I),BMU(I),
     * I=1,NBRES3)
      WRITE (6,540) GAM,BV,MOL,ECHEM,RHOP,B1(1),B2(1),BN(1),XLPRI,XPROP
      IF(NPRC.NE.0) WRITE(6,800) AUP,ADWN
      IF(NWFR.LT.0) WRITE(6,810) VISLYR,DELYR
      IF(NWFR.GT.0) WRITE(6,820) (AMUV(I),AMU(I),I=1,NWFR)
      IF (NBR.EQ.0) GO TO 110
      WRITE (6,690)
      DO 100 I=1,NBR
      WRITE (6,700) I,B1(I),B2(I),BN(I),ZR(I)
  100 CONTINUE
  110 WRITE (6,550)
      WRITE (6,470) (ITIT(I),I=1,20)
      IPRT=1
      IF (NPRO.EQ.1) WRITE (6,670)
      IF (NPRO.EQ.2) WRITE (6,630)
      IF (NPRO.EQ.3) WRITE (6,610)
      IF(NDSK.EQ.0) GO TO 129
      REWIND 8
      IF(NDSK.LT.2) GO TO 123
C
C     RESTART OF MOST RECENT CASE
C
  121 READ(8,END=122) MDSKID,MDSKDT,MPRC,MDIM,MDIM2
      IF(MDSKID.EQ.NDSKID.AND.MDSKDT.EQ.NDSKDT) GO TO 125
      GO TO 121
  122 WRITE(6,790) NDSKID,NDSK
      CALL EXIT
  123 IF(NDSKID.LE.1) GO TO 129
```

```
  124 READ(8,END=122) MDSKID
      IF(MDSKID.NE.99999) GO TO 124
      BACKSPACE 8
      GO TO 129
  125 BACKSPACE 8
      CALL REFIT(0,1)
      IEND=MDIM
      IF(MPRC.EQ.1) IEND=MDIM2
      READ(8)  MDSKID,NSTEP,NPRC,NDIM,NDIM2,NDT,NI,NP,NF,NDIM1,NBC,
     *         NPRT,NBRT,SSTART,PDIA,PMAX,PMAXA,STMAX,AKGMAX,
     *         DELTAX,DELTBX,DTI,TIME,RDOT,SIG,XBB,XLPRI,XPROP,
     *         PPMAX,(GS1(I,NF),GS2(I,NF),GS3(I,NF),I=1,IEND)
      TPRT=TIME
      NNPRC=NPRC
      SAVE(4)=XB
      BACKSPACE 8
      GO TO 130
  129 CALL BAR1
      NF=1
C
CCC   PRINT
C
  130 NDIMM=NDIM
      IF(NPRC.EQ.1) NDIMM=NDIM2
      CALL BAR2 (1,NDIMM,NF,XB)
      IF(NBRV.EQ.0) GO TO 135
      IF(P(1).GE.PDIA) NBC(1)=1
C
CCCC  PRINT AT BURNOUT AND AT EXTREMA OF PRESSURE
C
C     COMPUTE SOUND VELOCITY AND MACH NO.
  135 AO=DSQRT(G*GAM*P(NDIM)/RHO(NDIM)/(1.D0-BV*RHO(NDIM)))
      AMACH=DABS(U(NDIM)-VB)/AO
C
CCC   COMPUTE PROJECTILE ACCELERATION IN KILO-G:S
C
      IF (NBC(2).EQ.1) GO TO 138
      AKG=0.D0
      IF(NDT.GT.0) AKG=K(2,2)/G/1000.D0
  138 MAND=0
      IF(NNPRC.NE.NPRC) MAND=1
      NNPRC=NPRC
      IF (NNBC2.NE.NBC(2)) MAND=1
      NNBC2=NBC(2)
      IF (NBRT.GT.1) GO TO 150
      IF (CM.LE.1.D-10) GO TO 140
      GO TO 150
  140 NBRT=NBRT+1
      PBRBO=P(1)
      PBBO=P(NDIM)
```

```
      STRBO=SIG
      XBO=XPR
      TBO=TIME*1.D3
      VBO=DT1(2)/12.DO
      NSTBO=NSTEP
      AKGBO=AKG
      AMBO=AMACH
  150 IF (NBRT.EQ.1) MAND=1
      PBITA=0.DO
      DO 160 I=1,NDIM
      IF (P(I).GT.PBITA) PBITA=P(I)
  160 CONTINUE
      IPBIT=PBITA
      PBIT=IPBIT
      PMAXA=PMAX
      PMAX=PBIT
      IF (PBITA.GT.PPMAX) GO TO 170
      GO TO 180
  170 PPMAX=PBITA
      PBMAX=P(NDIM)
      PBRMAX=P(1)
      STRMAX=SIG
      XPMAX=XPR
      TPMAX=TIME*1.D3
      VPMAX=DT1(2)/12.DO
      NSPMAX=NSTEP
      ZPMAX=1.DO-DT1(3)/CMI
      AKGPM=AKG
      AMPM=AMACH
  180 DPMAXA=P(1)-P(NDIM)
      IF(DPMAXA.LT.DPMAX) GO TO 182
      DPMAX=DPMAXA
      PBPMX=P(NDIM)
      PBRPMX=P(1)
      STRPMX=SIG
      XPPMX=XPR
      TPPMX=TIME*1.D3
      VPPMX=DT1(2)/12.DO
      NPMX=NSTEP
      ZPMX=1.DO-DT1(3)/CMI
      AKGPMX=AKG
      AMPMX=AMACH
  182 IF(DPMAXA.GE.DPMIN) GO TO 184
      DPMIN=DPMAXA
      PBPMY=P(NDIM)
      PBRPMY=P(1)
      STRPMY=SIG
      YPPMY=XPR
      TPPMY=TIME*1.D3
      VPPMY=DT1(2)/12.DO
```

```
      NPMY=NSTEP
      ZPMY=1.D0-DT1(3)/CMI
      AKGPMY=AKG
      AMPMY=AMACH
  184 IF (AKG.GT.AKGMAX) GO TO 190
      GO TO 200
  190 AKGMAX=AKG
      PBRAM=P(1)
      PBAM=P(NDIM)
      STRAM=SIG
      XAM=XPR
      TAM=TIME*1.D3
      VPAM=DT1(2)/12.D0
      ZAM=1.D0-DT1(3)/CMI
      NSAM=NSTEP
      AMAM=AMACH
  200 TAME=TIME*1.D3
      IF (NBC(2).EQ.1) GO TO 205
      VELF=DT1(2)/12.D0
      Z=1.D0-DT1(3)/CMI
  205 IF (SIG.GT.STMAX) GO TO 210
      GO TO 220
  210 ASTMAX=AKG
      PBRSTM=P(1)
      PBSTM=P(NDIM)
      STMAX=SIG
      XSTM=XPR
      TSTM=TIME*1.D3
      VPSTM=DT1(2)/12.D0
      ZSTM=1.D0-DT1(3)/CMI
      NSTM=NSTEP
      AMSTM=AMACH
  220 IF (NPRO.EQ.4) GO TO 310
      IF (NSTEP.GT.NCY1) NPRO=0
      IF (NSTEP.GT.NCY2) NPRO=NPROI
      IF (DABS(TIME-TPRT).LE.1.D-10) GO TO 230
      IF (MAND.EQ.0) GO TO 335
  230 LINE=LINE+NDIM+18
      IF (LINE.LT.55) GO TO 240
      LINE=NDIM+18
      IF (NPRO.GT.0) GO TO 240
      WRITE (6,550)
C
C     COMPUTE SEGMENT VOLUME
C
  240 FAC=AB*XB*DELTAX
      SEGW(1)=FAC*.5D0*RHO(1)
      SEGW(NDIM)=FAC*.5D0*RHO(NDIM)
      SEGE(1)=FAC*.5D0*RHO(1)*E(1)
      SEGE(NDIM)=FAC*.5D0*RHO(NDIM)*E(NDIM)
```

```
      SEGKE(1)=FAC*.5D0*RHO(1)*U(1)**2/(2.D0*G)
      SEGKE(NDIM)=FAC*.5D0*RHO(NDIM)*U(NDIM)**2/(2.D0*G)
      TSEGW=SEGW(1)+SEGW(NDIM)
      TSEGIE=SEGE(1)+SEGE(NDIM)
      TSEGKE=SEGKE(1)+SEGKE(NDIM)
      KDIM=NDIM-1
      DO 250 KK=2,KDIM
C
CCC   SEGMENT MASS
C
      SEGW(KK)=FAC*RHO(KK)
      TSEGW=TSEGW+SEGW(KK)
C
CCC   SEGMENT INTERNAL ENERGY
C
      SEGE(KK)=FAC*RHO(KK)*E(KK)
      TSEGIE=TSEGIE+SEGE(KK)
C
CCC   SEGMENT KINETIC ENERGY
C
      SEGKE(KK)=FAC*RHO(KK)*U(KK)**2/(2.D0*G)
      TSEGKE=TSEGKE+SEGKE(KK)
  250 CONTINUE
C
CCC   CHEMICAL AND KINETIC ENERGY OF UNBURNT PROPELLANT
C
      CHEUPR=CM*ECHEM
      AKEUPR=CM*VPR**2/(2.D0*G)
C
CCC   PROJECTILE KINETIC ENERGY
C
      PRKE=PRM*VPR**2/(2.D0*G)
C
CCC   TOTAL MASS
C
      TMASS=TSEGW+CM
      CTDEL=100.D0*(1.D0-TMASS/CTOT)
      IF(DABS(CTDEL).GT.DABS(CTDELM)) CTDELM=CTDEL
C
CCC   TOTAL ENERGY
C
      TENG=TSEGIE+TSEGKE+CHEUPR+AKEUPR+PRKE
      ETDEL=100.D0*(1.D0-TENG/ETOT)
      IF(DABS(ETDEL).GT.DABS(ETDELM)) ETDELM=ETDEL
C
CCC   COMPUTE PERCENTAGE ENERGY BALANCE
C
      PIE=TSEGIE*100.D0/TENG
      PKE=TSEGKE*100.D0/TENG
      PCUPR=CHEUPR*100.D0/TENG
```

```
      PKEUPR=AKEUPR*100.D0/TENG
      PPRKE=PRKE*100.D0/TENG
      PITT=PIE+PKE+PCUPR+PKEUPR+PPRKE
      BIT=DABS(U(NDIM)-VB)
      BIT=BIT/DSQRT(G*GAM*P(NDIM)/RHO(NDIM)/(1.D0-BV*RHO(NDIM)))
      DO 260 I=1,NDIM
  260 POS(I)=DFLOAT(I-1)*XB*DELTAX
      IF(NPRC.EQ.0) GO TO 266
      DO  265 I=NDIM1,NDIM2
  265 POS(I)=XB+DFLOAT(I-NDIM1)*XBB*DELTBX
C
C     CHECK OUTPUT OPTIONS
C
  266 IF (NPRO.GT.0) GO TO 270
C
C     PROFILE DISTRIBUTION OUTPUT
C
      WRITE (6,560) NSTEP,TAME,DT1,RDOT,SIG,BIT,AKG,RESAIR,RESOB
      WRITE (6,570) TMASS,TSEGW,CM
      WRITE (6,580) TENG,TSEGIE,TSEGKE,CHEUPR,AKEUPR,PRKE,PITT,PIE,PKE,
     1   PCUPR,PKEUPR,PPRKE
      WRITE (6,590)
      WRITE (6,600) (POS(I),RHO(I),P(I),T(I),U(I),SEGKE(I),SEGE(I),
     1   SEGW(I),I=1,NDIM)
      IF(NPRC.EQ.1) WRITE(6,601) (POS(I),RHO(I),P(I),U(I),I=NDIM1,NDIM2)
      GO TO 310
  270 IF (NPRO.EQ.1) GO TO 290
      IF (NPRO.EQ.2) GO TO 280
C
C     ONE LINE IB DATA + ( ENERGY PRINTED
C
      PBR=P(1)/1000.D0
      PBASE=P(NDIM)/1000.D0
      STRESS=SIG/1000.D0
      WRITE (6,620) TAME,XPR,VELF,AKG,PBR,PBASE,STRESS,Z,DT1(4),RDOT,BIT
     1   ,PIE,PKE,PCUPR,PKEUPR,PPRKE,NSTEP
      GO TO 300
C
C     ONE LINE ENERGY TRAJECTORY
C
  280 WRITE (6,640) TAME,XPR,VELF,TENG,TSEGIE,TSEGKE,CHEUPR,AKEUPR,PRKE,
     1   PITT,PIE,PKE,PCUPR,PKEUPR,PPRKE
      GO TO 300
  290 WRITE (6,680) TAME,XPR,VELF,P(1),P(NDIM),DT1(3),Z,DT1(4),DT1(5),
     1   RDOT,SIG,BIT,NSTEP
C
C     PRINT TITLE LINE AFTER 50 TIME STEPS
C
  300 IPRT=IPRT+1
      IF (IPRT.LT.50) GO TO 310
```

```
      WRITE (6,550)
      WRITE (6,470) (ITIT(I),I=1,20)
      IF (NPRO.EQ.1) WRITE (6,670)
      IF (NPRO.EQ.2) WRITE (6,630)
      IF (NPRO.EQ.3) WRITE (6,610)
      IPRT=1
  310 IF (DABS(TIME-TPRT).GT.1.D-10) GO TO 320
      NPRT=NPRT+1
      TPRT=TPRT+TNOM
  320 IF (IDEAL.EQ.0) GO TO 330
C
C     CHECK MACH NUMBER OF REACTION PRODUCTS IN IDEAL CASE
C
      IF (NPRO.GT.0.OR.NBC(2).EQ.1) GO TO 330
      IF (BIT.GE.1.D0) WRITE (6,660)
C
C     CHOOSE TIME STEP
C
  330 IF(NDSK.NE.1.AND.NDSK.NE.3) GO TO 335
      IEND=NDIM
      IF(NPRC.EQ.1) IEND=NDIM2
      WRITE(8) NDSKID,NSTEP,NPRC,NDIM,NDIM2,NDT,NI,NP,NF,NDIM1,NBC,
     *         NPRT,NBPT,SSTART,PDIA,PMAX,PMAXA,STMAX,AKGMAX,
     *         DELTAX,DELTBX,DT1,TIME,RDOT,SIG,XBB,XLPRI,XPROP,
     *         PPMAX,(GS1(I,NF),GS2(I,NF),GS3(I,NF),I=1,IEND)
  335 IF (NEXIT.EQ.1) GO TO 420
      DT=TNOM
C
C     COURANT STABILITY CONDITION FOR FLOW
C
      IER=0
      CALL BAR4 (TSET,XB,XBB,SAFE,IER)
      IF (IER.EQ.1) GO TO 440
      IF (TSET.LT.DT) DT=TSET
      BIT=DABS(XB*QDRF(NDIM)/GS3(NDIM,NF))
      IF (BIT*DT.GT.0.1D0) DT=0.1D0/BIT
      IF (SIG*DT.GT.1.D0) DT=1.D0/SIG
      IF (TIME+DT.GT.TSTOP) DT=TSTOP-TIME
      IF (VPR.LT.1.D-10) GO TO 340
      BIT=(XSTOP-XPR)/VPR
      IF (DT.GT.BIT) DT=BIT
  340 IF (TIME+DT.GT.TPRT) DT=TPRT-TIME
  350 IF(NBC(2).EQ.1) GO TO 365
      CALL GETK (1)
      XBBP=XBB
      DO 360 I=1,4
      SAVE(I)=DT1(I)
  360 DT1(I)=DT1(I)+K(I,1)*DT
      XBB=XPR-XB+XIB+XLPRI
      SAVE(5)=VB
```

```
      IF (CM.LT.1.D-10) CM=0.DO
      IF(CM.LT.1.D-10) SAVE(3)=0.DO
      IF (XB.GT.XPR+XPROP-1.D-6) XB=XPR+XPROP
  365 CALL BAR3 (1,SAVE(4),VB,XBBP,VPR,DT)
      IF(NBC(1).EQ.0) CALL BREECH (SAVE(4),XB,DT)
      IF(NBC(1).EQ.1) CALL OUTFLO(1,SAVE(4),XB,DT,1.DO)
      IF(NPRC.EQ.1) CALL BASEPR(XBBP,DT)
      IER2=0
      IF(NBC(2).EQ.0) CALL BASE (SAVE(4),XB,XBBP,DT,IER2)
      IF(NBC(2).EQ.1) CALL OUTFLO(NDIM,SAVE(4),XB,DT,-1.DO)
      IF (IER2.EQ.1) GO TO 420
      IF (NCJ.EQ.1) GO TO 390
      IF(NPRC.EQ.1) CALL SIGCHK
      IF(NWFR.NE.0.AND.NPRC.EQ.1) CALL VELCHK
      IF(NBC(2).EQ.1) GO TO 366
      CALL GETK (2)
  366 CALL BAR3 (0,XB,VB,XBB,VPR,DT)
      IF(NBC(2).EQ.1) GO TO 375
      DO 370 I=1,4
  370 DT1(I)=SAVE(I)+0.5DO*(K(I,1)+K(I,2))*DT
      XBB=XPR-XB+XIB+XLPRI
      IF (CM.LT.1.D-10) CM=0.DO
      IF(CM.LT.1.D-10) SAVE(3)=0.DO
      IF (XB.GT.XPR+XPROP-1.D-6) XB=XPR+XPROP
  375 IF(NBC(1).EQ.0) CALL BREECH (SAVE(4),XB,DT)
      IF(NBC(1).EQ.1) CALL OUTFLO(1,SAVE(4),XB,DT,1.DO)
      IF(NPRC.EQ.1) CALL BASEPR(XBBP,DT)
      IF(NBC(2).EQ.0) CALL BASE (SAVE(4),XB,XBBP,DT,IER2)
      IF(NBC(2).EQ.1) CALL OUTFLO(NDIM,SAVE(4),XB,DT,-1.DO)
      IF (IER2.EQ.1) GO TO 420
      IF (NCJ.EQ.1) GO TO 390
      IF(NPRC.EQ.1) CALL SIGCHK
      IF(NWFR.NE.0.AND.NPRC.EQ.1) CALL VELCHK
      QLOS=(QDRF(1)+QDRF(NDIM))*0.5DO
      NDIML=NDIM-1
      DO 376 I=2,NDIML
      QLOS=QLOS+QDRF(I)
  376 CONTINUE
      QLOSS=QLOS*AB*DT*XB*DELTAX+QLOSS
      RESLS1=RESLS1+(RESAIR+RESOB)*AB*DT*VPR
      IF (NPRC.EQ.0) GO TO 378
      RESLS=(U(NDIM1)*QDRF(NDIM1)+U(NDIM2)*QDRF(NDIM2))*0.5DO
      NDIML=NDIM1+1
      NDIMR=NDIM2-1
      DO 377 I=NDIML,NDIMR
      RESLS=RESLS+U(I)*QDRF(I)
  377 CONTINUE
      RESLS2=RESLS2-RESLS*AB*DT*XDB*DELTAX
      GO TO 379
  378 RESLS2=RESLS2-PRPRES*AB*DT*XBB*VPR
```

```
  379 IF(NBC(2).EQ.0) CALL REFIT (1,NF)
      NSTEP=NSTEP+1
      TIME=TIME+DT
C
CC    TEST FOR TERMINATION
C
      IF (NSTOP.EQ.99999) GO TO 380
      IF (NSTEP.GE.NSTOP) NEXIT=1
  380 IF (TIME.GE.TSTOP) NEXIT=1
      IF(NBC(2).EQ.1) GO TO 383
      IF (XPR.LT.XSTOP-1.D-3) GO TO 385
      IF(NMUZBL.EQ.0) GO TO 384
      CM=0.D0
      VPR=0.D0
      T1MZ=TAME
      P1MZ=P(1)
      PNMZ=P(NDIM)
      SIGMZ=SIG
      AMACHZ=AMACH
      NSTEPZ=NSTEP
      WRITE (6,885)
      XB=XPR+XIB+XLPRI
      SAVE(4)=XB
      VB=0.D0
      NPRC=0
      NBC(2)=1
  383 IF(P(1).GT.PBRF) GO TO 385
  384 NEXIT=1
  385 IF (NEXIT.EQ.1) TPRT=TIME
      GO TO 130
  390 NEXIT=1
      TPRT=TIME
      IF (NCJ.NE.1) GO TO 130
      NF=NI
      IF (INT.EQ.0) NF=NP
      DO 400 I=1,5
  400 DT1(I)=SAVE(I)
      RDOT=VR-VPR
      IF(NPRC.EQ.1) RDOT=VB-GS2(NDIM1,NF)/GS1(NDIM1,NF)
      XBB=XPR-XB+XI3+XLPRI
      GO TO 130
  410 CALL EXIT
C
CC    PRINT OUT SUMMARY DATA
C
  420 IF (PPMAX.GT.PSTI) GO TO 430
C
CC    SET MAX. PRESSURE TO INITIAL CONDITIONS
C
      STRMAX=PSTI
```

```
      PBMAX=PSTI
      PBRMAX=PSTI
      PPMAX=PSTI
      AMPM=0.D0
      ZPMAX=0.D0
      VPMAX=0.D0
      XPMAX=0.D0
      TPMAX=0.D0
      NSPMAX=0
      AKGPM=PSTI*AB/((PRM+CMI)*1000.D0)
  430 WRITE (6,710)
      WRITE (6,470) (ITIT(I),I=1,20)
      WRITE (6,480) VELF,PPMAX,AKGMAX,STMAX
      WRITE (6,720)
      WRITE (6,770)
      WRITE (6,780) TBO,XBO,VBO,AKGBO,DUM,PBRBO,PBBO,STRBO,Z,AMBO,NSTBO
      WRITE (6,730)
      WRITE (6,770)
      WRITE (6,780) TPMAX,XPMAX,VPMAX,AKGPM,PPMAX,PBRMAX,PBMAX,STRMAX,
     1  ZPMAX,AMPM,NSPMAX
      WRITE (6,740)
      WRITE (6,770)
      WRITE (6,780) TAM,XAM,VPAM,AKGMAX,DUM,PBRAM,PBAM,STRAM,ZAM,AMAM,
     1  NSAM
      WRITE (6,750)
      WRITE (6,770)
      WRITE (6,780) TSTM,XSTM,VPSTM,ASTMAX,DUM,PBRSTM,PBSTM,STMAX,ZSTM,
     1  AMSTM,NSTM
      WRITE (6,760)
      WRITE (6,770)
      IF (NBC(2).EQ.1) GO TO 435
      P1MZ=P(1)
      PNMZ=P(NDIM)
      T1MZ=TAME
      SIGMZ=SIG
      AMACHZ=AMACH
      NSTEPZ=NSTEP
  435 WRITE (6,780) T1MZ,XPR,VELF,AKG,DUM,P1MZ,PNMZ,SIGMZ,Z,AMACHZ,
     *  NSTEPZ
      WRITE(6,764)
      WRITE(6,770)
      WRITE(6,780) TPPMX,XPPMX,VPPMX,AKGPMX,DUM,PBRPMX,PBPMX,STRPMX,
     *             ZPMX,AMPMX,NPMX
      WRITE(6,766)
      WRITE(6,770)
      WRITE(6,780) TPPMY,YPPMY,VPPMY,AKGPMY,DUM,PBPPMY,PBPMY,STRPMY,
     *             ZPMY,AMPMY,NPMY
      WRITE(6,990) CTDEL,CTDELM,ETDEL,ETDELM,OLOSS,RESLS1,RESLS2
      NENDOF=99999
      IF (NDSK.EQ.1.OR.NDSK.EQ.3) WRITE (8) NENDOF
```

```
C
CC      SWITCHES FOR VARYING C/M PARAMETRICALLY
C
        IF (NPAR.EQ.0) GO TO 20
        NPR=NPR+1
        IF (NPR.EQ.NPAR+1) GO TO 20
        NDSKID=NDSKID+1
        GO TO 60
  440   WRITE (6,510)
        GO TO 420
C
CC
C
  450   FORMAT (20A4)
  460   FORMAT (1H1,10X,42HSIMULATION OF END BURNING TRAVELING CHARGE,10X,
       120HVERSION (FEB 1 1980)//1X,20A4/)
  470   FORMAT (1X,20A4/)
  480   FORMAT (20X,15HMUZZLE VELOCITY,6X,5H(F/S),3X,F7.0/20X,16HMAXIMUM P
       1RESSURE,5X,5H(PSI),1X,F9.0/20X,26HMAXIMUM ACCELERATION (K-G),3X,F7
       2.0/20X,14HMAXIMUM STRESS,7X,5H(PSI),1X,F9.0/)
  490   FORMAT(16I5)
  495   FORMAT(4I5,6F10.0)
  500   FORMAT (8F10.0)
  510   FORMAT (31H NEG. SQUARE ROOT ERROR IN BAR4)
  520   FORMAT(10X,12HCONTROL DATA//
       *42H IDEAL BURN RATE LAW                                ,I5/
       *42H CONTINUUM MODEL OF UNREACTED PROPELLANT             ,I5/
       *42H PRINT OPTION                                       ,I5/
       *42H DISC READ/WRITE PARAMETER                          ,I5/
       *42H PROBLEM ID FOR DISC READ/WRITE                     ,I5/
       *42H INTEGRATION STEP FOR DISC RESTART                  ,I5/
       *42H NUMBER OF PARAMETRIC CASES                         ,I5/
       *42H NUMBER OF BURN RATE PARAMETERS                     ,I5/
       *42H PROPELLANT WALL FRICTION PARAMETER                 ,I5/
       *42H NUMBER OF ENTRIES IN PROJECTILE BORE               /
       *42H              RESISTANCE TABLE                      ,I5/
       *42H INDICATOR FOR AIR RESISTANCE                       ,I5/
       *42H NUMBER OF ENTRIES IN OBTURATOR FRICTION            /
       *42H                                TABLE               ,I5/
       *42H WALL HEAT LOSS OPTION                              ,I5/
       *42H TUBE BLOW-DOWN OPTION                              ,I5/
       *42H RECOILLESS TUBE OPTION                             ,I5//)
  525   FORMAT(5X,22HINTEGRATION PARAMETERS//
       *42H MAXIMUM NUMBER OF STEPS                            ,I5/
       *42H MAXIMUM NUMBER OF MESH POINTS                      ,I5/
       *42H FIRST CYCLE TO BE PRINTED WITH AXIAL               /
       *42H                        DISTRIBUTION               ,I5/
       *42H LAST CYCLE TO BE PRINTED WITH AXIAL                /
       *42H                        DISTRIBUTION               ,I5/
       *42H TIME STEP FOR LOGOUT CYCLE (MSEC)                  ,F10.4/
```

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     *42H PROBLEM TERMINATION TIME (MSEC)                    ,F10.3/
     *42H MAXIMUM PROJECTILE TRAVEL (INS)                    ,F10.3/
     *42H STABILITY SAFETY FACTOR                            ,F10.3/
     *42H MINIMUM MESH SIZE (INS)                            ,F10.3//)
  530 FORMAT (11X,37HTUBE,PROJECTILE AND CHARGE PROPERTIES//42H BORE DIA
     1METER (INS)                    ,F10.3/42H INITIAL POSITION OF RE
     2AR FACE OF                ,/42H PROPELLANT (INS)
     3,F10.3/42H PROJECTILE MASS (LBM)                       ,F10.5/42H CHA
     4RGE MASS (LBM)                    ,F10.3/42H INITIAL PRESSURE
     5(PSI)                    ,F10.0/41H MAXIMUM PRESSURE IN UNREACTED P
     6ROPELLANT,/42H (PSI), IF IDEAL=2                       ,F10.0/42H
     7 MAXIMUM MACH NUMBER OF REACTION PRODUCTS ,F10.3/42H MAXIMUM ACCEL
     8ERATION OF PROJECTILE(GRAV) ,F10.0/42H          C/M
     9                ,F10.2/42H CHAMBER VOLUME (IN**3)
     A,F10.5/42H LOADING DENSITY(LB/IN**3)                   ,F7.5/42H SHOT
     B-START PRESSURE  (PSI)                ,F10.0//)
  540 FORMAT (11X,24HPROPERTIES OF PROPELLANT//42H RATIO OF SPECIFIC HEA
     1TS (-)                    ,F10.3/42H COVOLUME (IN**3/LBM)
     2       ,F10.3/42H MOLECULAR WEIGHT (LBM/LBMOL)                 ,F10.3/4
     33H CHEMICAL ENERGY OF PROPELLANT (LBF-IN/LBM),F10.0/42H DENSITY OF
     4 PROPELLANT (LBM/IN**3)                ,F10.4/42H BURNING RATE ADDITIVE CO
     5NSTANT (IN/SEC)  ,F10.4/42H BURNING RATE PRE-EXPONENTIAL FACTOR
     6   /42H                  (IN/SEC-PSI**BN)                  ,F10.6/42H BURNIN
     7G RATE EXPONENT (-)                    ,F10.4/42H TC GRAIN LENGTH (IN)
     8                ,F10.3/42H LENGTH BREECH TO PROJECTILE BASE (
     9IN)     ,F10.3//)
  550 FORMAT (1H1)
  560 FORMAT (1HO,10X,16HSOLUTION AT STEP,I5,5X,12HTIME(MSEC) =,F7.3//50
     1H PROJECTILE TRAVEL(INS)                               ,F10.3/50H PROJ
     2ECTILE VELOCITY(INS/SEC)                          ,F10.0/50H UNREACTED
     3PROPELLANT MASS(LBM)                         ,F10.4/50H POSITION OF PROP
     4ELLANT BOUNDARY(INS)                    ,F10.3/50H VELOCITY OF PROPELLANT
     5 BOUNDARY(INS/SEC)              ,F10.0/50H PROPELLANT REGRESSION RATE(I
     6NS/SEC)                    ,F10.2/50H STRESS ON UNREACTED SIDE OF FLAME(
     7PSI)                ,F10.0/50H MACH NUMBER OF REACTION PRODUCTS(-)
     8          ,F10.3/
     *50H PROJECTILE ACCELERATION (KG)                                ,F10.3/
     *50H RESISTANCE DUE TO SHOCKED AIR (PSI)                         ,F10.0/
     *50H RESISTANCE DUE TO OBTURATOR (PSI)                           ,F10.0/)
  570 FORMAT (11X,5HTOTAL,6X,3HGAS,3X,19HUNBURNED PROPELLANT/10H MASS(LB
     1):,F8.4,1X,F8.4,6X,F8.4/)
  580 FORMAT (20X,5HTOTAL,6X,8HINTERNAL,2X,11HGAS KINETIC,1X,13HUNBURNT
     1PROP.,1X,17HCHE UNBURNT PROP.,1X,2HKE,2X,13HPROJECTILE KE/1X,14HEN
     2ERGY(IN-LB):,3E12.6,3X,E12.6,6X,E12.6,5X,E12.6/2X,13H %  ENERGY  :
     3,3(4X,F7.2),7X,F7.2,2(10X,F7.2))
  590 FORMAT (40X,32HDISTRIBUTIONS OF STATE VARIABLES//40H POSITION DENS
     1ITY  PRESSURE  TEMPERATURE,40H VELOCITY KINETIC ENERGY INTERNAL EN
     2ERGY,3X,4HMASS/3X,3HINS,4X,8HLB/IN**3,3X,3HPSI,8X,5HDEG-R,6X,3HI/S
     3,6X,5HIN/LB,11X,5HIN/LB,10X,2HLB/)
  600 FORMAT (1X,F7.3,1X,F9.6,1X,F9.0,2X,F6.0,4X,F9.1,1X,E12.6,4X,E12.6,
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   13X,F9.6)
601 FORMAT(1X,F7.3,F10.6,1X,F9.0,12X,F9.1)
610 FORMAT (75X,21(1H*),8H( ENERGY,18(1H*)/3X,4HTIME,3X,12HTRAVEL  VEL
   1.,2X,6HACCEL.,3X,15HPRESSURE (KPSI),7X,1HZ,6X,2HXB,5X,1HR,5X,14HMA
   2CH  INTERNAL,4X,3HGAS,4X,27HUNBURNT  UNBURNT PROJECTILE,1X,4HSTEP/
   34X,2HMS,6X,2HIN,4X,3HF/S,4X,3HK-G,3X,6HBREECH,2X,4HBASE,3X,6HSTRES
   4S10X,2HIN,6X,3HI/S,20X,7HKINETIC,2X,25HCHEMICAL KINETIC  KINETIC/)
620 FORMAT (1X,F7.2,F8.2,F7.0,F7.1,2F7.2,F8.2,F6.3,F7.1,F8.0,F7.3,5(1X
   1,F8.3),2X,I5)
630 FORMAT (5H TIME,4X,6HTRAVEL,2X,8HVELOCITY,2X,43H-------------------
   1--ENERGY(IN-LB)----------,36H----------------------****** % ENERGY,
   218H******************/4H  MS,7X,2HIN,7X,3HF/S,5X,5HTOTAL,5X,8HINTE
   3RNAL,3X,6HGAS KE,6X,7HUBP CHE,4X,6HUBP KE,3X,7HPROJ.KE,3X,11HTOTAL
   4  INT.,25H GAS KE UB CE UB KE PR KE/)
640 FORMAT (1X,F7.2,F9.2,F8.1,6G11.5,6F6.1)
660 FORMAT (54H WARNING. IDEAL CASE REQUIRES STRONG DEFLAGRATION WAVE)
670 FORMAT (49H  TIME    TRAVEL   VELOCITY PBREECH P BASE P-MASS,40H
   1   Z        XB     XBDOT     R    P-STRESS,13H MACH    STEP/)
680 FORMAT (1X,F8.3,F9.2,F8.1,2F8.0,F7.3,F7.4,F7.1,2F8.0,F9.0,F7.3,I6)
690 FORMAT (11X,23HVARIABLE BURN RATE DATA/43H STEP  INTERCEPT COEFFIC
   1IENT EXPONENT    Z)
700 FORMAT (1X,I3,3X,F8.3,1X,G12.6,1X,F8.4,2X,F7.4)
710 FORMAT (1HI,27X,14HSUMMARY OUTPUT//)
720 FORMAT (31X,7HBURNOUT)
730 FORMAT (27X,15HMAXIMUM PRESSURE)
740 FORMAT (27X,20HMAXIMUM ACCELERATION)
750 FORMAT (27X,14HMAXIMUM STRESS)
760 FORMAT (32X,6HMUZZLE)
764 FORMAT(26X,21HMAX. FORWARD GRADIENT)
766 FORMAT(26X,21HMAX. REVERSE GRADIENT)
770 FORMAT (3X,4HTIME,3X,22HTRAVEL VELOCITY  ACCEL,1X,7(1H-),14HPRESSU
   1RE (PSI),12(1H-),3X,1HZ,5X,10HMACH NSTEP/4X,2HMS,6X,2HIN,5X,3HF/S,
   26X,2HKG,4X,21HMAXIMUM BREECH   BASE,4X,6HSTRESS,11X,2HNO)
780 FORMAT (1X,F7.2,F8.2,1X,F8.1,F8.2,3F8.0,F9.0,F7.4,F7.4,I5/)
790 FORMAT(22H RESTART WITH NDSKID =,I10,11H AND NDSK =,I10,
   *20H FAILS. TERMINATING.)
800 FORMAT(
   *42H COMPRESSION WAVE SPEED IN PROPELLANT      /
   *42H                          (IN/SEC)         ,F10.0/
   *42H EXPANSION WAVE SPEED IN PROPELLANT        /
   *42H                          (IN/SEC)         ,F10.0//)
810 FORMAT(
   *42H VISCOSITY OF LUBRICATING FILM             /
   *42H          (LBM/IN-SEC)                     ,E10.3/
   *42H THICKNESS OF LUBRICATING FILM (INS)       ,F10.4//)
820 FORMAT(1HO, 8X,36HFRICTION BETWEEN PROPELLANT AND TUBE//
   *44H         VELOCITY          FRICTION COEFFICIENT/
   *44H         (IN/SEC)                (-)          /
   *(1H ,F15.1,10X,F10.3))
830 FORMAT(1HO, 8X,35HRESISTIVE PRESSURE DUE TO OBTURATOR//
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     *44H          TRAVEL            RESISTIVE PRESSURE /
     *44H           (INS)                 (PSI)         /
     *(1H ,F15.3,10X,F10.0))
  840 FORMAT(
     *42H RATIO OF SPECIFIC HEATS OF AIR (-)          ,F10.4/
     *42H PRESSURE OF AIR IN BARREL (PSI)             ,F10.3/
     *42H TEMPERATURE OF AIR IN BARREL (DEG.R)        ,F10.1/
     *42H MOLECULAR WEIGHT OF AIR IN BARREL           /
     *42H                 (LBM/LBMOL)                 ,F10.4//)
  845 FORMAT(
     *42H MASS OF PROJECTILE AHEAD OF OBTURATOR       /
     *42H                                  (LBM)      ,F10.5/
     *42H LENGTH OF OBTURATOR (INS)                   ,F10.3/
     *42H POISSON RATIO OF PROJECTILE (-)             ,F10.3//
     * 8X,35HFRICTION BETWEEN OBTURATOR AND TUBE//
     *44H          VELOCITY          FRICTION COEFFICIENT/
     *44H           (IN/SEC)               (-)         /
     *(1H ,F15.1,10X,F10.3))
  850 FORMAT(
     *42H TEMPERATURE OF TUBE WALL (DEG.R)            ,F10.1/
     *42H COEFFICIENT IN HEAT TRANSFER CORRELATION    ,F10.4/
     *42H VISCOSITY OF GAS (LBM/IN-SEC)               ,E10.3/
     *42H PRANDTL NUMBER OF GAS (-)                   ,F10.3//)
  860 FORMAT(
     *42H BLOWDOWN TERMINATION PRESSURE (PSI)         ,F10.1/
     *42H DISCHARGE COEFFICIENT FOR MUZZLE (-)        ,F10.3//)
  870 FORMAT(
     *42H THROAT AREA OF BREECH NOZZLE (IN**2)        ,F10.4/
     *42H DISCHARGE COEFFICIENT OF BREECH NOZZLE(-),F10.3/
     *42H RUPTURE PRESSURE FOR BREECH DIAPHRAGM       /
     *42H                                  (PSI)      ,F10.1//)
  880 FORMAT(1HO,24X,21HGLOBAL BALANCE CHECKS/
     *20X,20HFINAL MASS DEFECT(%),4X,F10.3/
     *20X,22HMAXIMUM MASS DEFECT(%),2X,F10.3/
     *20X,22HFINAL ENERGY DEFECT(%),2X,F10.3/
     *20X,24HMAXIMUM ENERGY DEFECT(%),F10.3///
     *20X,'TOTAL HEAT LOSS TO TUBE (LBF-IN)',F10.0/
     *20X,'LOSS DUE TO PROJECTILE RESISTANCE (LBF-IN)',F10.0/
     *20X,'LOSS DUE TO PROPELLANT RESISTANCE (LBF-IN)',F10.0/)
  885 FORMAT(' MUZZLE EXIT HAS OCCURRED,  PROCEEDING WITH BLOWDOWN',
     1    ' ANALYSIS')
      END
```

```
      SUBROUTINE BAR1
C
CCC   INITIALIZES COMPUTATIONAL ARRAYS. THIS ROUTINE MUST BE CALLED
CCC   AT OUTSET OF INTEGRATION PROCEDURE.
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL, MACH
C
      COMMON /BARA/ GS1(100,3), GS2(100,3), GS3(100,3)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1   CV, ZR(20)
      COMMON /BARF/ XPR, VPR, CM, XB, VB, AB, PRM, RDOT, SIG, SSTART
      COMMON /BARG/ XBB,DELTBX,NPRC,NDIM1,NDIM2
      COMMON /BARH/ AUP,ADWN,VISLYR,DELYR,AMUV(10),AMU(10),DB,
     *              BRX(10),BR(10),AIRGAM,AIRPO,AIRTO,AIRMW,TWALO,
     *              PBRF,CDMUZ,ASBR,CDBR,PDIA,CHTW,DRHO(2)
      COMMON /BARI/ NWFR,NBRES1,NBRES2,NHTW,NMUZBL,NBRV
      COMMON /BARCJ2/ PST, SIGMAX, MACH, APMAX, XI1, XI2, NCJ, IDEAL
      COMMON /BRVAR/ XLPPI, XPROP,NBR
      DATA G /386.1600/
C
      NDT=0
      DO 10 I=1,900
   10 GS1(I,1)=0.D0
      CALL REFIT (0,1)
      IF (CM.GT.0.D0.AND.IDEAL.EQ.0) GO TO 30
      IF (CM.GT.0.D0.AND.IDEAL.EQ.2) GO TO 80
   20 RHOST=1.D0/(BV+(GAM-1.D0)*ECHEM/PST)
      DUB=0.D0
      ESTA=ECHEM
      SIG=PST
      RDOT=0.D0
      GO TO 50
   30 CALL BURN (PST,RDOT,RDOTPR)
      X1=2.D0*G/RDOT*(PST/RHOP*GAM/(GAM-1.D0)-RDOT*RDOT/G)
      X2=RDOT*RDOT-2.D0*G*(ECHEM+BV*PST/(GAM-1.D0)+PST/RHOP)
      X3=X1*X1-4.D0*X2
      IF (X3.LT.0.D0) GO TO 70
      X3=DSQRT(X3)
      IF (X1.LT.0.D0) X3=-X3
      X3=(-X1+X3)*0.5D0
      UB=RDOT+VPR-X3
      RHOST=RDOT*RHOP/X3
      GO TO 105
   40 ESTA=PST*(1.D0-BV*RHOST)/RHOST/(GAM-1.D0)
      SIG=PST+RHOP/G*RDOT*RDOT*(RHOP/RHOST-1.D0)
      DUB=UB/DFLOAT(NDIM-1)
   50 A=RHOST*XB
```

```
      DO 60 J=1,3
      DO 60 I=1,NDIM
      GS1(I,J)=A
      B=DFLOAT(I-1)*DUB
      GS2(I,J)=A*B
   60 GS3(I,J)=A*(ESTA+B*R*0.5D0/G)
      VB=RDOT
      IF(NPRC.EQ.0) RETURN
C
C     DISTRIBUTIONS IN CONTINUUM PROPELLANT
C
      IF(NWFR.GT.0) GO TO 62
      A=RNOM(SIG)
      XLPRI=CM/A/AB
      XPPOP=XLPRI+XB
      XBB=XLPRI
      DO 61 J=1,3
      DO 61 I=NDIM1,NDIM2
      GS1(I,J)=A
      GS3(I,J)=SIG
   61 CONTINUE
      RETURN
   62 ITER=0
      SUMA=CM/AB
      XBBL=0.D0
      XBBH=XBB
   63 XBB=0.5D0*(XBBL+XBBH)
      BIT=-4.D0/D3*AMU(1)*DELTBX*XBB
      SUM=0.D0
      DO 65 I=NDIM1,NDIM2
      SIGIJ=SIG*DEXP(BIT*DFLOAT(I-NDIM1))
      GS3(I,1)=SIGIJ
      RIJ=RNOM(SIGIJ)
      GS1(I,1)=RIJ
      FAC=1.D0
      IF((I-NDIM1)*(I-NDIM2).EQ.0) FAC=0.5D0
      SUM=SUM+RIJ*FAC
   65 CONTINUE
      SUM=SUM*DELTBX
      IF(DABS(SUM-SUMA/XBB).LT.1.D-6) GO TO 68
      ITER=ITER+1
      IF(ITER.LT.50) GO TO 66
      WRITE(6,120)
      CALL EXIT
   66 IF(SUM.GT.SUMA/XBB) GO TO 67
      XBBL=XBB
      GO TO 63
   67 XBBH=XBB
      GO TO 63
   68 DO 69 J=2,3
```

```
      DO 69 I=NDIM1,NDIM2
      GS1(I,J)=GS1(I,1)
      GS3(I,J)=GS3(I,1)
   69 CONTINUE
      XLPRI=XBB
      XPROP=XLPRI+XB
      RETURN
   70 WRITE (6,110)
      CALL EXIT
   80 TEST1=SIGMAX
      IF (APMAX.LT.1.D-10) GO TO 90
      TEST1A=RESP(XPR,VPR,AFAX)
      TEST1=APMAX*(PRM+CM)/AB+TEST1A+AFAX*APMAX*G
      IF (SIGMAX.LT.1.D-10) GO TO 90
      IF (SIGMAX.LT.TEST1) TEST1=SIGMAX
   90 IF (PST.GE.TEST1-1.D-10) GO TO 20
      BIT=G/RHOP/RHOP/2.DO
      NWAY=1
      BIT=G/RHOP/RHOP/2.DO
      BIT=BIT*(TEST1-PST)*(TEST1+(GAM+1.DO)/(GAM-1.DO)*PST)
      BIT=BIT/(ECHEM+(BV-1.DO/RHOP)*PST/(GAM-1.DO))
      RHOST=RHOP/(1.DO+(TEST1-PST)*G/RHOP/BIT)
  100 RDOT=DSQRT(BIT)
      UB=RDOT*(1.DO-RHOP/RHOST)
      IF (NWAY.EQ.2) GO TO 40
C
C     CHECK MACH NUMBER
C
  105 IF (MACH.LT.1.D-10) GO TO 40
      BITM=DABS(UB-RDOT)/DSQRT(G*GAM*PST/RHOST/(1.DO-BV*RHOST))
      IF (BITM.LE.MACH) GO TO 40
      IF (MACH.LT.1.DO) GO TO 107
      BITN=ECHEM+TEST1/RHOP-BV*PST/(GAM-1.DO)*(1.DO+(GAM-1.DO)*GAM*MACH
     1   *MACH)
      BITNN=RHOP*BV
      BITNN=BITNN*BITNN-1.DO
      BITL=BITNN*(GAM*PST*MACH/RHOP)**2*(MACH*MACH+2.DO/(GAM-1.DO))
  106 DIS=BITN*BITN-BITL
      IF (DIS.LT.0.DO) GO TO 70
      BIT=(BITN-DSQRT(DIS))*G/BITNN
      RHOST=1.DO/(BV+GAM*MACH*MACH*G*PST/RHOP/RHOP/BIT)
      NWAY=2
      GO TO 100
  107 BITNN=1.DO-BV*RHOP
      BITMM=1.DO+GAM*MACH*MACH
      BITN=ECHEM+BITNN*BITMM*PST/RHOP
      BITL=BITNN*GAM*PST*MACH/RHOP
      BITL=2.DO/(GAM-1.DO)*BITL*BITL*(1.DO+(GAM-1.DO)/2.DO*MACH*MACH)
      BITNN=BITNN*BITNN
      GO TO 106
```

```
C
  110 FORMAT (43H NEGATIVE DISCRIMINANT IN BAR1. TERMINATING)
  120 FORMAT(53H EXCESSIVE ITERATIONS TO DETERMINE PROPELLANT LENGTH.)
      END
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```
      SUBROUTINE BAR2 (I,J,K,XBP)
C
CCC   TRANSFORMS COMPUTATIONAL ARRAYS FOR LEVEL K INTO CUSTOMARY
CCC   STATE VARIABLES FOR MESH RANGE I TO J
CCC   ARGUMENT XBP IS THE POSITION OF THE PROPELLANT AT LEVEL K.
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
      COMMON /BARA/ GS1(100,3), GS2(100,3), GS3(100,3)
      COMMON /BARB/ RHO(100), P(100), E(100), T(100), U(100), ETA(100)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1   CV, ZR(20)
      COMMON /BARG/ XBB,DELTBX,NPPC,NDIM1,NDIM2
      DATA  G /386.160D0/
C
      II=I
      JJ=J
      IF(I.GT.NDIM) GO TO 15
      IF(J.GT.NDIM) JJ=NDIM
      DO 10 L=I,JJ
      RHO(L)=GS1(L,K)/XBP
      U(L)=GS2(L,K)/GS1(L,K)
      E(L)=GS3(L,K)/GS1(L,K)-U(L)*U(L)*0.5D0/G
      P(L)=(GAM-1.D0)*RHO(L)*E(L)/(1.D0-BV*RHO(L))
   10 T(L)=E(L)/CV
      IF(J.LE.NDIM) RETURN
      II=NDIM1
      JJ=J
   15 IF(NPRC.EQ.0) GO TO 20
      DO 16 L=II,JJ
      RHO(L)=GS1(L,K)
      U(L)=GS2(L,K)
   16 P(L)=GS3(L,K)
      RETURN
   20 WRITE (6,30) I,J,NDIM
      CALL EXIT
C
   30 FORMAT (40H BAR2 CALLED WITH ILLEGAL ARGUMENTS, I =,I10,4H J =,I10
     1,7H NDIM =,I10,12H TERMINATING)
      END
```

```
      SUBROUTINE BAR3 (JNT,XBP,VBP,XBBP,VBBP,DT)
C
C     UPDATES COMPUTATIONAL ARRAYS USING A TWO-LEVEL EXPLICIT
C     MARCHING SCHEME. ROUTINE MUST BE CALLED TWICE PER UPDATE
C     CYCLE, ONCE WITH JNT = 1 (PREDICTOR STEP) AND THEN WITH JNT
C     = 0 (CORRECTOR STEP). ONLY THE INTERIOR MESH POINT VALUES ARE
C     UPDATED; THE BOUNDARY VALUES ARE TREATED EXTERNALLY.
C     ARGUMENTS XBP AND VBP ARE THE POSITION AND VELOCITY OF THE
C     PROPELLANT AT THE PRESENT LEVEL.
C     ARGUMENT XBBP IS THE LENGTH OF THE PROPELLANT AT THE PRESENT
C     LEVEL AND VBBP IS THE VELOCITY OF THE PROJECTILE.
C     ARGUMENT DT IS THE TIME STEP OVER WHICH THE SOLUTION IS BEING
C     INCREMENTED.
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON /BARA/ GS1(100,3), GS2(100,3), GS3(100,3)
      COMMON /BAR3/ RHO(100), P(100), E(100), T(100), U(100), ETA(100)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARD/ DXMIN, DELTAX
      COMMON /BARG/ XBB,DELTBX,NPRC,NDIM1,NDIM2
      COMMON /BARI/ NWFR,NBRES1,NBRES2,NHTW,NMUZBL,NBRV
      COMMON /BARJ/ QORF(100),PRPRES
C
      DIMENSION GS(100,3,3)
      EQUIVALENCE (GS(1,1,1),GS1(1,1))
      DATA G /386.1600/
C
      NI=MOD(NDT,3)+1
      NF=MOD(NDT+1,3)+1
      NP=MOD(NDT+2,3)+1
      NDIMM=NDIM
      IF(NPRC.EQ.1) NDIMM=NDIM2
      INT=MOD(NDT+1,2)
      DO 5 I=1,NDIMM
    5 QORF(I)=0.D0
      CALL BAR2 (1,NDIMM,NI,XBP)
      IF(NWFR.NE.0.AND.NPRC.EQ.1) CALL WFR
      IF(NHTW.NE.0) CALL HTW
      DO 10 I=1,NDIM
   10 ETA(I)=U(I)-DFLOAT(I-1)*VBP*DELTAX
      DX=DELTAX*XBP
      DELTX=DELTAX
      IL=2
      IR=NDIM-1
      IF(NPRC.EQ.0) GO TO 15
      DO 14 I=NDIM1,NDIM2
   14 ETA(I)=U(I)-(DFLOAT(I-NDIM1)*DELTBX*(VBBP-VBP)+VBP)
   15 IF (INT.EQ.0) GO TO 20
      IP=1
```

```
      IM=0
      GO TO 25
   20 IP=0
      IM=1
   25 DO 70 I=IL,IR
      IF(I.GT.NDIM) GO TO 26
      I2=I+IP
      I1=I-IM
      GS1(I,NF)=(GS1(I1,NI)*ETA(I1)-GS1(I2,NI)*ETA(I2))/DX
      GS2(I,NF)=(GS2(I1,NI)*ETA(I1)-GS2(I2,NI)*ETA(I2))/DX+
     *(P(I1)-P(I2))/DELTX*G
      GS3(I,NF)=(GS3(I1,NI)*ETA(I1)-GS3(I2,NI)*ETA(I2))/DX+(P(I1)*U(I1)
     1   -P(I2)*U(I2))/DELTAX-QORF(I)*XBP
      GO TO 35
   26 X=RHO(I)
      JL=I
      IF(ETA(I).GT.0.D0) JL=I-1
      JR=JL+1
      XD=-X*(U(I+1)-U(I-1))/DX2
      GS1(I,NF)=-ETA(I)*(RHO(JR)-RHO(JL))/DX+XD
      AP=DSQR(X,XD,P(I))
      GS3(I,NF)=-ETA(I)*(P(JR)-P(JL))/DX+AP*AP/G*XD
      DPDX=(P(I-1)-P(I+1))/DX2
      IF(DABS(QORF(I)).LT.1.D-10) GO TO 34
      IF(DABS(U(I)).GT.1.D-3) GO TO 33
C
C     CHECK FOR LOCKING DUE TO FRICTION
C
      IF(DPDX.GT.0.D0) GO TO 32
      DPDX=DPDX-QORF(I)
      IF(DPDX.GT.0.D0) DPDX=0.D0
      GO TO 34
   32 DPDX=DPDX+QORF(I)
      IF(DPDX.LT.0.D0) DPDX=0.D0
      GO TO 34
   33 DPDX=DPDX+QORF(I)
   34 GS2(I,NF)=-ETA(I)*(U(JR)-U(JL))/DX+G*DPDX/X
   35 IF (INT.EQ.0) GO TO 50
C
C     PREDICTOR STEP
C
      DO 40 K=1,3
   40 GS(I,NF,K)=GS(I,NI,K)+GS(I,NF,K)*DT
      GO TO 70
C
C     CORRECTOR STEP
C
   50 DO 60 K=1,3
   60 GS(I,NF,K)=0.5D0*(GS(I,NP,K)+GS(I,NI,K)+GS(I,NF,K)*DT)
   70 CONTINUE
```

```
      IF(NPRC.EQ.0.OR.IR.GT.NDIM) GO TO 80
      IL=NDIM1+1
      IR=NDIM2-1
      DX=DELTSX*XBBP
      DX2=2.00*DX
      GO TO 25
   80 NDT=NDT+1
      RETURN
      END
```

```
      SUBROUTINE BAR4 (TSET,XB,XBB,SAFE,IER)
C
CCC   COURANT STABILITY CONDITION FOR  FLOW.
CCCCC
CCCCC TSET IS THE MAXIMUM VALUE WHICH THE TIME STEP MAY HAVE FOR
CCCCC STABLE INTEGRATION OF THE BALANCE EQUATIONS FOR THE FLOW.
CCCCC ARGUMENT XB IS THE PRESENT VALUE OF THE GAS/PROPELLANT
CCCCC INTERFACE AND XBB IS THE PRESENT LENGTH OF THE PROPELLANT.
CCCCC ARGUMENT SAFE IS A VALUE LARGER THAN ONE AND IS USED TO RESTRICT
CCCCC THE TIME STEP MORE STRINGENTLY THAN THE BASIC COURANT CONDITION.
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
C
      COMMON /BARB/ RHO(100), P(100), E(100), T(100), U(100), ETA(100)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARD/ DXMIN, DELTAX
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1   CV, ZR(20)
      COMMON /BARG/ YBB,DELTBX,NPRC,NDIM1,NDIM2
      DATA G /386.16D0/
C
      TSET=0.D0
      DO 10 I=1,NDIM
      TCON=(GAM*G*P(I)/RHO(I)/(1.D0-BV*RHO(I)))
      IF (TCON.LT.0.D0) GO TO 20
      C=DSQRT(GAM*G*P(I)/RHO(I)/(1.D0-BV*RHO(I)))
      BIT=DABS(U(I)-ETA(I))+C
      IF (BIT.GT.TSET) TSET=BIT
   10 CONTINUE
      TSET=XB*DELTAX/TSET/SAFE
      IF(NPRC.EQ.0) RETURN
      TSET1=0.D0
      DO 15 I=NDIM1,NDIM2
      AP=DSDR(RHO(I),1.D0,P(I))
      APA=DSDR(RHO(I),-1.D0,P(I))
      IF(APA.GT.AP) AP=APA
      BIT=DABS(U(I)-ETA(I))+AP
      IF(BIT.GT.TSET1) TSET1=BIT
   15 CONTINUE
      TSET1=XBB*DELTBX/TSET1/SAFE
      IF(TSET1.LT.TSET) TSET=TSET1
      RETURN
   20 I=1
      IER=1
      RETURN
      END
```

```
      SUBROUTINE BASE (XBP,XBF,XBBP,DT,IER)
C
CCC   UPDATES BOUNDARY VALUES AT GAS/PROPELLANT INTERFACE
CCC
CCC   ARGUMENTS XBP AND XBF ARE RESPECTIVELY THE POSITIONS OF THE
CCC   INTERFACE AT THE PRESENT AND FUTURE LEVELS. ARGUMENT DT IS
CCC   THE TIME STEP OVER WHICH THE SOLUTION IS BEING UPDATED.
CCC   ARGUMENT XBBP IS THE LENGTH OF THE PROPELLANT AT THE PRESENT LEVEL
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL, MACH
C
      COMMON /BARA/ GS1(100,3), GS2(100,3), GS3(100,3)
      COMMON /BARB/ RHO(100), P(100), E(100), T(100), U(100), ETA(100)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1  CV, ZR(20)
      COMMON /BARF/ XPP, VPR, CM, XB, VB, AB, PRM, RDOT, SIG, SSTART
      COMMON /BARG/ XBB,DELTBX,NPRC,NDIM1,NDIM2
      COMMON /BARH/ AUP,ADWN,VISLYR,DELYR,AMUV(10),AMU(10),DB,
     *              BRX(10),BR(10),AIRGAM,AIRPO,AIRTO,AIRMW,TWALO,
     *              BBRF,CDMUZ,ASBR,CDBR,PDIA,CHTW,DRHO(2)
      COMMON /BARCJ2/ PST, SIGMAX, MACH, APMAX, XI1, XI2, NCJ, IDEAL
      COMMON /BARJ/ QDRF(100),PRPRES
      COMMON /CJCO/ ICJ
      DATA R, G /18550.D0, 386.16D0/
C
      NCJ=0
      NSUP=0
      JDEAL=0
      ITEPP=0
      VVPP=VPR
      RHOPP=RHOP
      IF(NPRC.EQ.0) GO TO 5
      VVPR=U(NDIM1)
      CALL CARAC (NDIM1,VVPR,SSIG,DSDV,XBP,XBBP,DT,+1.D0,C,DRHO(1))
      DIFFB=(VB-VVPR)*DT/XBBP/DELTBX
      DIFFA=1.D0-DIFFB
      P2=P(NDIM1)*DIFFA+P(NDIM1+1)*DIFFB
      RHO2=RHO(NDIM1)*DIFFA+RHO(NDIM1+1)*DIFFB
      C=0.5D0*(C+DSDR(RHO2,DRHO(1),P2))
      C2P=C*C
      RHOPP=RHO2+(SSIG-P2)*G/C2P
      DRHO(1)=(RHOPP-RHO2)/DT
    5 IF (NDT.EQ.1) NOUT=0
      IF (CM.GT.0.D0) GO TO 20
      IF (NOUT.GE.10) GO TO 60
C
CC    BOUNDARY VALUE OF PRESSURE FROM SIMPLE WAVE ANALYSIS FOR FIVE
C     INTEGRATION CYCLES AFTER BURNOUT
```

```
C
      IF (NOUT.NE.0) GO TO 10
      PST=P(NDIM)
      RHOST=RHO(NDIM)
      UST=U(NDIM)
   10 UNF=VVPR
      WPR=VVPR-UST
      TP=(1.D0-(GAM-1.D0)/2.D0*WPR/DSQRT(G*GAM*PST*(1.D0-BV*RHOST)/RHOST
     1   ))
      IF (TP.LT.0.D0) GO TO 230
      PNF=PST*TP**(2.D0*GAM/(GAM-1.D0))
      NOUT=NOUT+1
      RHONF=1.D0/(BV+(PST/PNF)**(1.D0/GAM)*(1.D0/RHOST-BV))
      RDOT=0.D0
      SIG=PNF
      IF (PNF.GT..0500*PST) GO TO 200
      NCJ=1
      WRITE (6,270)
      RETURN
   20 IF (IDEAL.NE.1) GO TO 60
      IF (U(NDIM).GT.1.D-10) GO TO 50
C
CC    IDEAL BURN RATE LAW (IDEAL=0)
C
      UNF=0.D0
      PNF=PST
      BIT=XI2-VVPR*VVPR/2.D0/G
      IF (BIT.GT.0.D0) GO TO 30
      WRITE (6,250)
      NCJ=1
      RETURN
   30 RDOT=XI1*VVPR/BIT
      IF(RDOT.LE.1.D-10) GO TO 70
      RHONF=RHOPP*RDOT/(RDOT+VVPR)
      IF (RHONF.LT.1.D0/BV) GO TO 40
      IF (MACH.GT.1.D-10) GO TO 50
      WRITE (6,260)
      NCJ=1
      RETURN
C
CC    CHECK MACH NUMBER OF REACTION PRODUCTS
C
   40 IF (MACH.LE.1.D-10) GO TO 190
      BITARG=G*GAM*PNF/RHONF/(1.D0-BV*RHONF)
      IF(BITARG.LE.0.D0) GO TO 50
      BIT=DABS(UNF-VVPR-RDOT)/DSQRT(BITARG)
      IF (BIT.GE.1.D0) GO TO 50
      IF (BIT.LE.MACH) GO TO 189
   50 JDEAL=1
   60 ITER=0
```

```
      UNF=U(NDIM)
      IF (CM.LE.0.D0) UNF=VVPR
      CALL CARAC (NDIM,UNF,PNF,XA,XBP,XBBP,DT,-1.D0,C,0.D0)
      IF (CM.GT.0.D0) GO TO 80
   70 RDOT=0.D0
      SIG=PNF
      C=0.5D0*(C+DSQRT(G*GAM*P(NDIM)/RHO(NDIM)/(1.D0-BV*RHO(NDIM))))
      C2=C*C
      RHONF=RHO(NDIM)+G/C2*(PNF-P(NDIM)+QORF(NDIM)*(GAM-1.D0)/
     *       (1.D0-BV*RHO(NDIM))*DT)
      GO TO 192
   80 YB=PNF-UNF*XA
   90 IF (JDEAL.EQ.0) GO TO 100
      NRBIT=0
      IF(MACH.GE.1.D0) GO TO 95
      BIT1=(PNF-YB)/XA-VVPR
      BIT2=1.D0-BV*RHOPP
      BIT3=4.D0*BIT2*MACH*MACH*GAM*G/RHOPP
      TMP=BIT1**2+BIT3*PNF
      IF (TMP.LT.0.D0) GO TO 230
      BIT4=DSQRT(TMP)
      RDOT=(BIT1+BIT4)/BIT2*0.5D0
      RDOTPR=(1.D0/XA+0.5D0/BIT4*(2.D0/XA*BIT1+BIT3))/BIT2*0.5D0
      GO TO 140
   95 IF(IDEAL.EQ.2) GO TO 96
      NCJ=1
      WRITE(6,240)
      RETURN
   96 RBIT=G/RHOPP*(TEST1-(1.D0+GAM*MACH*MACH)*PNF)/(BV*RHOPP-1.D0)
      IF (PNF.GT.TEST1) PNF=TEST1
      IF(RBIT.GT.0.D0) GO TO 97
      NRBIT=NRBIT+1
      PNF=0.95D0*PNF
      IF(NRBIT.LT.10) GO TO 96
      WRITE(6,245)
      RETURN
   97 RDOT=DSQRT(RBIT)
      RDOTPR=-0.5D0*G/RHOPP/RDOT*(1.D0+GAM*MACH*MACH)/(BV*RHOP-1.D0)
      PSI=GAM*G*MACH*MACH/RHOPP/RDOT/RDOT
      PSIPR=PSI*(1.D0-2.D0*PNF*RDOTPR/RDOT)
      PSI=RHOPP*(BV+PNF*PSI/RHOPP)
      NSUP=1
      GO TO 145
  100 IF (IDEAL.EQ.2) GO TO 110
      IF(PNF.GT.0.D0) GO TO 105
      IF(MACH.GT.1.D-10) GO TO 50
      NCJ=1
      RETURN
  105 CALL BURN (PNF,RDOT,RDOTPR)
      GO TO 140
```

```
C
C     IDEAL CASE (IDEAL=2)
C
  110 IF (APMAX.LT.1.D-10) GO TO 120
      TEST1A=RESP(XPR,VPR,AFAX)
      TEST1=APMAX*(PRM+CM)/AB+TEST1A+AFAX*APMAX*G
      IF (SIGMAX.LT.1.D-10) GO TO 130
      IF (SIGMAX.LT.TEST1) TEST1=SIGMAX
      GO TO 130
  120 TEST1=SIGMAX
  130 BIT1=TEST1+(GAM+1.D0)/(GAM-1.D0)*PNF
      BIT2=XA*VVPR+YB-PNF
      BIT2=BIT2*0.5D0/XA/RHOPP
      BIT3=ECHEM+(BV-1.D0/RHOPP)*PNF/(GAM-1.D0)
      RDOT=BIT1*BIT2/BIT3
      RDOTPR=(GAM+1.D0)/(GAM-1.D0)*BIT2/BIT3-BIT1/BIT3*0.5D0/XA/RHOPP
     1   -BIT1*BIT2/BIT3/BIT3*(BV-1.D0/RHOPP)/(GAM-1.D0)
  140 CONTINUE
      IF(RDOT.LE.1.D-10) GO TO 175
      PSI=(VVPR+RDOT-(PNF-YB)/XA)
      PSIPR=(RDOTPR-1.D0/XA)*RDOT-PSI*RDOTPR
      PSI=PSI/RDOT
      PSIPR=PSIPR/RDOT/RDOT
  145 FP1=(GAM/(GAM-1.D0)*PSI-1.D0-BV*RHOPP/(GAM-1.D0))/RHOPP
      FP2=PSI-1.D0
      FP=PNF*FP1-ECHEM+RDOT*RDOT*FP2*FP2/2.D0/G
      FPPR=FP1+PNF*GAM/(GAM-1.D0)/RHOPP*PSIPR+RDOT*RDOTPR*FP2*FP2/G
     *  +RDOT
     1   *RDOT*FP2*PSIPR/G
      IF (DABS(FP).LT.1.D0) GO TO 170
      IF (ITER.LT.50) GO TO 150
      WRITE (6,280)
      GO TO 170
  150 PNF=PNF-FP/FPPR
      ITER=ITER+1
      IF (JDEAL.EQ.1) GO TO 90
      GO TO 100
  170 IF (IDEAL.EQ.0) GO TO 180
      IF (RDOT.GT.0.D0) GO TO 190
  175 UNF=VVPR
      CALL CARAC (NDIM,UNF,PNF,XA,XBP,XBBP,DT,-1.D0,C,0.D0)
      GO TO 70
  180 IF(NSUP.EQ.0) GO TO 185
      RHONF=RHOPP/PSI
      UNF=VVPR+(1.D0-PSI)*RDOT
      GO TO 188
  185 UNF=(PNF-YB)/XA
      RHONF=RDOT*RHOPP/(VVPR+RDOT-UNF)
  188 IF (IDEAL.NE.1.AND.JDEAL.EQ.0) GO TO 40
  189 IF(IDEAL.NE.0) GO TO 190
```

```
      IF(RHONF.LT.0.9D0/BV.AND.PNF.GT.0.D0 ) GO TO 190
      IF(JDEAL.EQ.1) GO TO 190
      IF(MACH.GT.1.D-10.AND.MACH.LT.1.D0) GO TO  50
      WRITE(6,310)
      RETURN
  190 SIG=PNF+RHOPP*RDOT*RDOT/G*(RHOPP/RHONF-1.D0)
  192 IF(NPRC.EQ.0) GO TO 200
      DSSIG=SIG-SSIG
      IF(DABS(DSSIG).LT.1.D-3) GO TO 200
      IF(IIERP.LT.50) GO TO 195
      WRITE(6,300)
      GO TO 200
  195 ITERP=ITERP+1
      VVPR=VVPR+DSSIG/DSDV
      RHOPP=RHOPP+DSSIG*G/C2P
      RHOPPP=RNOM(SIG)
      IF(RHOPP.LT.RHOPPP) RHOPP=RHOPPP
      SSIG=SIG
      GO TO 20
  200 ENF=PNF*(1.D0-BV*RHONF)/(GAM-1.D0)/RHONF
  210 U(NDIM)=UNF
      P(NDIM)=PNF
      E(NDIM)=ENF
      T(NDIM)=ENF/CV
      RHO(NDIM)=RHONF
      BIT=XBF*RHONF
      VB=VVPR+RDOT
      GS1(NDIM,NF)=BIT
      GS2(NDIM,NF)=BIT*UNF
      GS3(NDIM,NF)=BIT*(ENF+UNF*UNF*0.5D0/G)
      IF(NPRC.EQ.0) GO TO 220
      U(NDIM1)=VVPR
      P(NDIM1)=SIG
      RHO(NDIM1)=RHOPP
      GS1(NDIM1,NF)=RHOPP
      GS2(NDIM1,NF)=VVPR
      GS3(NDIM1,NF)=SIG
  220 RETURN
C   SET PNF ERROR CONDITION
  230 WRITE (6,290)
      IER=1
      GO TO 220
C
  240 FORMAT(42H ILLEGAL PARAMETERS FOR SUPERSONIC BURNING)
  245 FORMAT(47H NEGATIVE SQUARE ROOT DURING SUPERSONIC BURNING)
  250 FORMAT (39H IDEAL BURN RATE INFINITE. TERMINATING.)
  260 FORMAT (54H IDEAL BURN RATE PRODUCES DENSITY LESS THAN RECIPROCAL/
     126H OF COVOLUME. TERMINATING.)
  270 FORMAT (53H1BASE PRESSURE DROPS TO LESS THAN 5% OF INITIAL VALUE/3
     12H FOLLOWING BURNOUT. TERMINATING.)
```

```
280 FORMAT (28H FAILURE TO CONVERGE IN BASE)
290 FORMAT (29H PNF FUNCTION IN BASE<0--STOP)
300 FORMAT(54H FAILURE TO CONVERGE IN BASE WITH CONTINUUM PROPELLANT)
310 FORMAT(50H COVOLUME LIMIT APPROACHED WITH MEASURED BURN RATE)
    END
```

```
      SUBROUTINE BASEPR (XBBP,DT)
C
C     UPDATES BOUNDARY VALUES AT INTERFACE BETWEEN PROJECTILE
C     BASE AND CONTINUUM PROPELLANT.
C
C     ARGUMENT XBBP IS THE LENGTH OF THE PROPELLANT AT THE PRESENT
C     LEVEL AND DT IS THE TIME STEP.
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
C
      COMMON /BARA/ GS1(100,3), GS2(100,3), GS3(100,3)
      COMMON /BARB/ RHO(100), P(100), E(100), T(100), U(100), ETA(100)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1   CV, ZR(20)
      COMMON /BARF/ XPR, VPR, CM, XB, VB, AB, PRM, RDOT, SIG, SSTART
      COMMON /BARG/ XBB,DELTBX,NPRC,NDIM1,NDIM2
      COMMON /BARH/ AUP,ADWN,VISLYR,DELYR,AMUV(10),AMU(10),DB,
     *              BRX(10),BR(10),AIRGAM,AIRPO,AIRTO,AIRMW,TWALO,
     *              BBRF,CDMUZ,ASBR,CDBR,PDIA,CHTW,DRHO(2)
      DATA G /386.16D0/
C
      CALL CARAC(NDIM2,VPR,PNF,DPDU,0.D0,XBBP,DT,-1.D0,C,DRHO(2))
      U(NDIM2)=VPR
      C=0.5D0*(C+DSDR(RHO(NDIM2),DRHO(2),P(NDIM2)))
      C2=C*C
      RHOPP=RNOM(PNF)
      BIT=RHO(NDIM2)
      RHO(NDIM2)=RHO(NDIM2)+G/C2*(PNF-P(NDIM2))
      IF(RHO(NDIM2).LT.RHOPP) RHO(NDIM2)=RHOPP
      DRHO(2)=(RHO(NDIM2)-BIT)/DT
      P(NDIM2)=PNF
      GS1(NDIM2,NF)=RHO(NDIM2)
      GS2(NDIM2,NF)=VPR
      GS3(NDIM2,NF)=PNF
      RETURN
      END
```

```
      SUBROUTINE BREECH (XBP,XBF,DT)
C
CCCCCC
      UPDATES BOUNDARY VALUES  AT BREECH OF TUBE WHEN IMPERMEABLE

      ARGUMENTS XBP AND XBF ARE RESPECTIVELY THE POSITIONS OF THE
      PROPELLANT AT THE PRESENT AND FUTURE LEVELS.
      ARGUMENT DT IS THE TIME STEP OVER WHICH THE SOLUTION IS BEING
      INCREMENTED.
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
C
      COMMON /BARA/ GS1(100,3), GS2(100,3), GS3(100,3)
      COMMON /BARB/ RHO(100), P(100), E(100), T(100), U(100), ETA(100)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1   CV, ZR(20)
      COMMON /BARJ/ QORF(100),PRPRES
      DATA G /386.16D0/
C
      CALL CARAC (1,0.D0,PNF,DPDU,XBP,0.D0,DT,1.D0,C,0.D0)
      U(1)=0.D0
      C=0.5D0*(C+DSQRT(G*GAM*P(1)/RHO(1)/(1.D0-BV*RHO(1))))
      C2=C*C
      RHO(1)=RHO(1)+G/C2*(PNF-P(1)+QORF(1)*(GAM-1.D0)/
     *        (1.D0-BV*RHO(1))*DT)
      P(1)=PNF
      E(1)=P(1)/RHO(1)/(GAM-1.D0)*(1.D0-BV*RHO(1))
      T(1)=E(1)/CV
      BIT=XBF*RHO(1)
      GS1(1,NF)=BIT
      GS2(1,NF)=0.D0
      GS3(1,NF)=BIT*E(1)
      RETURN
      END
```

```
      SUBROUTINE BURN (PNF,RDOT,RDOTPR)
C
C     CALCULATES REGRESSION RATE (RDOT) AND DERIVATIVE WITH RESPECT
C     TO PRESSURE (RDOTPR) AS A FUNCTION OF PRESSURE (PNF) FOR
C     EXPONENTIAL BURN RATE LAW (IDEAL=0)
C     SUBROUTINE ALLOWS FOR MULTIPLE BURN RATE FUNCTIONS WHICH CHANGE
C     AS THE TC GRAIN BURNS.
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
C
      COMMON /BARF/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1   CV, ZR(20)
      COMMON /BARF/ XPP, VPR, CM, XB, DUM(5)
      COMMON /BRVAR/ XLPRI, XPROP,NBR
C
C     CHECK FOR VARIATION OF BURN RATE FUNCTION.
C
      IF (NBR.EQ.0) GO TO 20
      XLPR=XPROP+XPR-XB
C
C     COMPUTE FRACTION OF GRAIN MASS BURNED
C
      ZP=1.D0-CM/RHOP/XLPRI
      IF (ZP.LT.0.D0) GO TO 20
      DO 10 I=1,NBR
C
C     CHECK FOR FRACTION OF GRAIN MASS BURNED, IF EXCEEDED
C     CHANGE BURN RATE FUNCTION.
C
      IF (ZP.LT.ZR(I)) GO TO 30
   10 CONTINUE
      B1I=B1(NBR)
      B2I=B2(NBR)
      BNI=BN(NBR)
      GO TO 40
   20 B1I=B1(1)
      B2I=B2(1)
      BNI=BN(1)
      GO TO 40
   30 B1I=B1(I)
      B2I=B2(I)
      BNI=BN(I)
      I=1
   40 CONTINUE
      RDOT=B2I*PNF**BNI
      RDOTPR=RDOT*BNI/PNF
      RDOT=RDOT+B1I
      RETURN
      END
```

```
      SUBROUTINE CARAC (KK,UNF,PNF,DPDU,XBP,XBBP,DT,SGN,CR,DRHO)
C
C     USES CONDITIONS OF COMPATIBILITY ON ACOUSTIC CHARACTERISTICS
C     TO DETERMINE BOUNDARY VALUES OF PRESSURE (PNF) CONSISTENT
C     WITH VELOCITY (UNF) AT POINT KK. ALSO RETURNS DERIVATIVE
C     OF PRESSURE WITH RESPECT TO VELOCITY (DPDU), SPEED OF SOUND
C     (CR).
C     ARGUMENT XBP IS THE POSITION OF THE GAS/PROPELLANT INTERFACE
C     AT THE PRESENT LEVEL AND XBBP IS THE LENGTH OF THE PROPELLANT.
C     ARGUMENT DT IS THE TIME STEP OVER WHICH THE SOLUTION IS BEING
C     INCREMENTED.
C     ARGUMENT SGN IS SET EQUAL TO 1 FOR A LEFT HAND BOUNDARY AND -1
C     FOR A RIGHT HAND BOUNDARY. ARGUMENT DRHO IS THE TIME DERIVATIVE
C     OF THE PROPELLANT DENSITY.
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 I2, MOL,MACH
C
      COMMON /BARB/ RHO(100), P(100), E(100), T(100), U(100), ETA(100)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARD/ DXMIN, DELTAX
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1   CV, ZR(20)
      COMMON /BARF/ XPR, VPR, CM, XB, VB, AB, PRM, RDOT, SIG, SSTART
      COMMON /BARG/ XBB,DELTBX,NPRC,NDIM1,NDIM2
      COMMON /BARI/ NWFR,NBRES1,NBRES2,NHTW,NMUZBL,NBRV
      COMMON /BARJ/ QORF(100),PRPRES
      COMMON /BARCJ2/ PST, SIGMAX, MACH, APMAX, XI1, XI2, NCJ, IDEAL
      COMMON /LPRO/ NPRO
      DATA G /386.1500/
C
      UREL=0.D0
      IF(KK.EQ.1) GO TO 2
      IF(NPRC.EQ.1.AND.KK.EQ.NDIM2) GO TO 2
      UREL=UNF-VB
    2 IF(SGN.LT.0.D0) GO TO 4
      IA=KK
      IB=KK+1
      GO TO 10
    4 IA=KK-1
      IB=KK
   10 IF(KK.LE.NDIM) GO TO 20
      C=DSDR(RHO(KK),DRHO,P(KK))
      I2=DFLOAT(NDIM1)+(-(SGN-1.D0)/2.D0-DT*(-SGN*C+UREL)/XBBP)
     *   /DELTBX
      GO TO 25
   20 C=DSQRT(GAM*P(KK)*G/RHO(KK)/(1.D0-BV*RHO(KK)))
      SGM=SGN
      IF(DABS(DRHO).LE.1.D-10) GO TO 24
      SGM=DRHO
```

```
      IF(SGN.LT.0.D0) GO TO 22
      IA=IA-1
      IB=IB-1
      GO TO 24
   22 IA=IA+1
      IB=IB+1
   24 I2=1.D0+(-(SGM-1.D0)/2.D0-DT*(-SGN*C+UREL)/XBP)/DELTAX
   25 IF (I2.GE.DFLOAT(IA).AND.I2.LE.DFLOAT(IB)) GO TO 40
      IF (NPRO.GT.0) GO TO 30
      IF(IDEAL.GT.0.AND.MACH.LT.1.D-10) GO TO 30
      IF(IDEAL.GT.0.AND.MACH.GE.0.99D0) GO TO 30
      WRITE (6,50) KK,I2,C,UREL
   30 CONTINUE
      IF (I2.LT.DFLOAT(IA)) I2=DFLOAT(IA)
      IF (I2.GT.DFLOAT(IB)) I2=DFLOAT(IB)
   40 DIFFA=IB-I2
      DIFFB=I2-IA
      CR=C
      NB=NI
      IF (INT.EQ.0) NB=NP
      RHO1=RHO(KK)
      CALL BAR2 (IA,IB,NB,XBP)
      U2=U(IA)*DIFFA+U(IB)*DIFFB
      P2=P(IA)*DIFFA+P(IB)*DIFFB
      RHO2=RHO(IA)*DIFFA+RHO(IB)*DIFFB
      RHO2=0.5D0*(RHO2+RHO1)
      BIT=(QORF(KK)+DIFFA*QORF(IA)+DIFFB*QORF(IB))*DT*0.5D0
      IF(KK.GT.NDIM) GO TO 45
      BIT=-BIT*(GAM-1.D0)/(1.D0-BV*RHO2)
      C=0.5D0*(C+DSQRT(GAM*P2*G/RHO2/(1.D0-BV*RHO2)))
   44 PNF=P2-SGN*RHO2*C*(U2-UNF)/G+BIT
      DPDU=SGN*RHO2*C/G
      RETURN
   45 C=0.5D0*(C+DSQR(RHO2,DRHO,P2))
      IF(BIT*UNF.GT.0.D0) BIT=-BIT
      BIT=-SGN*C*BIT
      IF(NWFR.LE.0) GO TO 44
      BIT=BIT/P(KK)
      DPDU=SGN*RHO2*C/G/(1.D0-BIT)
      IF(DABS(BIT).LT.1.D-10) GO TO 46
      IF(DABS(U2).LT.1.D0.AND.DABS(UNF).LT.1.D0) GO TO 47
   46 PNF=P2/(1.D0-BIT)-DPDU*(U2-UNF)
      RETURN
   47 PNF=P(KK)
      RETURN
C
   50 FORMAT (40H CHARACTERISTIC INTERCEPT OUT OF RANGE. WARNING.,4G20.6
     1)
      END
```

```
      FUNCTION DSDR(X,XD,Y)
C
CCCC  COMPUTES SMALL AMPLITUDE WAVE SPEED IN SOLID PROPELLANT AS
      A FUNCTION OF DENSITY (X), RATE-OF-CHANGE-OF-DENSITY (XD), AND
      PRESSURE (Y)
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1   CV, ZR(20)
      COMMON /BARH/ AUP,ADWN,VISLYR,DELYR,AMUV(10),AMU(10),
     *              BRX(10),BR(10),AIRGAM,AIRPO,AIRTO,AIRMW,TWALO,
     *              PBRF,COMUZ,ASBR,CDBR,PDIA,CHTW,DRHO(2)
C
      DSDR=AUP*X/RHOP
      IF(XD.LT.0.D0) GO TO 10
      YN=SNOM(X)
      IF(Y.LT.YN-1.D-10) GO TO 10
      RETURN
   10 IF(ADWN.GT.DSDR) DSDR=ADWN
      RETURN
      END
```

```
      SUBROUTINE GETK (I)
C
C     DERIVATIVES OF PROPELLANT AND PROJECTILE MOTION
C
C     GETK IS CALLED TWICE PER UPDATE CYCLE WITH I=1 FOR THE PREDICTOR
C     STEP AND I=2 FOR THE CORRECTOR STEP.
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL, K
C
      COMMON /BARB/ RHO(100), P(100), E(100), T(100), U(100), ETA(100)
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1   CV, ZR(20)
      COMMON /BARF/ XPR, VPR, CM, XB, VB, AB, PRM, RDOT, SIG, SSTART
      COMMON /BARG/ XBB,DELTBX,NPRC,NDIM1,NDIM2
      COMMON /BARH2/ PRMB,ELB,ANU,BMUV(10),BMU(10),NBRES3
      COMMON /BARH4/ RESAIR,RESOB
      COMMON /BARJ/ QORF(100),PRPRES
      COMMON /BERG/ K(4,2)
      DATA G /386.160D0/
C
C     CHECK FOR SHOT-START CONDITION
C
      IF (SIG.GT.SSTART) GO TO 10
      IF(NBRES3.NE.0) GO TO 10
      K(1,I)=0.D0
      K(2,I)=0.D0
      GO TO 20
   10 K(1,I)=VPR
      IF(NBRES3.EQ.0) SSTART=0.D0
      IF(NPRC.EQ.1) GO TO 12
      SIGX=SIG-RESP(XPR,VPR,A)
      PRTOT=PRM+CM
      GO TO 14
   12 SIGX=P(NDIM2)-RESP(XPR,VPR,A)+PRPRES*XBB
      PRTOT=PRM
   14 IF(SIGX.LT.0.D0.AND.VPR.LE.0.D0) SIGX=0.D0
      K(2,I)=AB*G*SIGX/PRTOT/(1.D0+A*AB*G/PRTOT)
      RESOB=RESOB+A*K(2,I)
   20 K(3,I)=0.D0
      RHOPX=RHOP
      IF(NPRC.EQ.1) RHOPX=RHO(NDIM1)
      IF (CM.GT.0.D0) K(3,I)=-RDOT*AB*RHOPX
      K(4,I)=VB
      RETURN
      END
```

```
      SUBROUTINE HTW
C
C     HEAT LOSS TO TUBE WALL FROM GAS
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
C
      COMMON /BARB/ RHO(100), P(100), E(100), T(100), U(100), ETA(100)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARE/ GAM, RV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1  CV, ZR(20)
      COMMON /BARG/ XBB,DELTBX,NPRC,NDIM1,NDIM2
      COMMON /BARH/ AUP,ADWN,VISLYR,DELYR,AMUV(10),AMU(10),DB,
     *              BRX(10),BR(10),AIRGAM,AIRPO,AIRTO,AIRMW,TWALO,
     *              BRF,COMUZ,ASBR,CDBR,PDIA,CHTW,DRHO(2)
      COMMON /BARH3/ PRNO,VISG
      COMMON /BARJ/ QORF(100),PRPRES
C
      BIT=VISG*CV*GAM/PRNO**0.6D0
      DO 10 I=1,NDIM
      QORF(I)=0.D0
      IF(T(I).LT.TWALO) GO TO 10
      REY=RHO(I)*U(I)*DB/VISG
      REY=DABS(REY)
      QORF(I)=CHTW/DB/DB*(T(I)-TWALO)*REY**0.8D0*BIT
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OUTFLO(KK,XBP,XBF,DT,SGN)
C
C     BOUNDARY VALUES AT GAS PERMEABLE BOUNDARY VENTING TO
C     ATMOSPHERE.
C
C     ARGUMENT KK POINTS TO STORAGE LOCATION FOR BOUNDARY DATA.
C     XBP IS THE LENGTH OF THE GAS COLUMN AT THE PRESENT UPDATE
C     LEVEL AND XBF IS THE VALUE AT THE FUTURE UPDATE LEVEL. DT
C     IS THE TIME STEP THROUGH WHICH THE SOLUTION IS BEING ADVANCED.
C     SGN IS SET EQUAL TO 1 AT A LEFT HAND BOUNDARY AND -1 AT A RIGHT
C     HAND BOUNDARY.
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
C
      COMMON /BARA/ GS1(100,3), GS2(100,3), GS3(100,3)
      COMMON /BARB/ RHO(100), P(100), E(100), T(100), U(100), ETA(100)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARD/ DXMIN, DELTAX
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1  CV, ZR(20)
      COMMON /BARF/ XPR, VPR, CM, XB, VB, AB, PRM, RDOT, SIG, SSTART
      COMMON /BARH/ AUP,ADWN,VISLYR,DELYR,AMUV(10),AMU(10),DB,
     *              BRX(10),BR(10),AIRGAM,AIRPO,AIRTO,AIRMW,TWALO,
     *              PBRE,COMUZ,ASBR,COBR,PDIA,CHTW,DRHO(2)
      COMMON /BARJ/ QDRF(100),PRPRES
      DATA R,G /18550.D0,386.1600/
C
      UNF=U(KK)
      QDRFBT=QDRF(KK)*(GAM-1.D0)/(1.D0-BV*RHO(KK))*DT
      CALL CARAC(KK,UNF,PNF,DPDU,XBP,1.D0,DT,SGN,C,0.D0)
      NWAY=0
      IF(DABS(UNF/C).LT.1.D0) GO TO 50
C
C     SEARCH FOR SUPERSONIC EFFLUX
C
      SGN2=-SGN
      CALL CARAC(KK,UNF,PNF2,DPDU2,XBP,1.D0,DT,SGN2,C,SGN)
      SAVE=UNF
      UNF=UNF+(PNF2-PNF)/(DPDU-DPDU2)
      SAVEP=PNF
      PNF=PNF+DPDU*(UNF-SAVE)
      NWAY=1
   50 DIFFB=-SGN*UNF*DT/XBP/DELTAX
      IF(KK.EQ.NDIM) GO TO 54
      DIFFA=1.D0-DIFFB
      IA=1
      GO TO 56
   54 DIFFA=DIFFB
      DIFFB=1.D0-DIFFB
```

```
      IA=NDIM-1
   56 IB=IA+1
      P2=P(IA)*DIFFA+P(IB)*DIFFB
      RHO2=RHO(IA)*DIFFA+RHO(IB)*DIFFB
      C=0.5D0*(C+DSQRT(G*GAM*P2/RHO2/(1.D0-BV*RHO2)))
      GC2=G/C/C
   60 RHONF=RHO2+((PNF-P2)+QDRFBT)*GC2
      ENF=PNF*(1.D0-BV*RHONF)/(GAM-1.D0)/RHONF
      TNF=ENF/CV
      IF(NWAY.EQ.0) GO TO 100
      IF(NWAY.EQ.2) GO TO 110
      CBIT=DSQRT(G*GAM*PNF/RHONF/(1.D0-BV*RHONF))
      IF(DABS(UNF/CBIT).GE.1.D0) GO TO 104
   90 NWAY=0
      UNF=SAVE
      PNF=SAVEP
      GO TO 50
C
CC    SEARCH FOR SUBSONIC EFFLUX
C
  100 ITER=0
      LSET=0
      LHSET=0
      NWAY=2
  104 IF(KK.NE.1) GO TO 105
      CDB=CDBR
      AST=ASBR
      GO TO 110
  105 CDB=CDMUZ
      AST=AB
C
CC    QUASI-STEADY FLOW RATE
C
  110 BIT=1.D0+UNF*UNF/2.D0/G/ENF/GAM
      NEXT=0
  130 PS=PNF*BIT**(GAM/(GAM-1.D0))
      IF(NEXT.EQ.1) GO TO 140
      NEXT=1
      BIT=BIT-BV*(PS-PNF)/ENF/GAM
      GO TO 130
  140 TS=TNF*BIT
      BITP=BV*PS/R/TS*MOL
      CRIT=CDB*AST*PS*DSQRT(GAM*G*MOL/R/TS*(2.D0/(GAM+1.D0))**
     *     ((GAM+1.D0)/(GAM-1.D0)))
     *      *(1.D0-0.224D0*BITP+0.104D0*BITP*BITP)
  150 IF(NWAY.EQ.2) GO TO 155
      CRIT2=AB*RHONF*DABS(UNF)
      IF(CRIT2.LE.1.0100*CRIT) GO TO 190
      GO TO 90
  155 UNFA=UNF
```

```
      UNFB=-SGN*CRIT/AB/RHONF
      IF(DABS(UNFB-UNFA).LT.1.D-2*DABS(UNFA)) GO TO 190
      UNF=UNFB
      ITER=ITER+1
      IF(ITER.GT.50) GO TO 200
      IF(UNF.GT.UNFA) GO TO 160
      UNFH=UNFA
      LHSET=1
      GO TO 170
  160 UNFL=UNFA
      LSET=1
  170 IF(LSET*LHSET.EQ.0) GO TO 180
      UNF=0.500*(UNFH+UNFL)
      PNF=PNF+DPDU*(UNF-UNFA)
      GO TO 60
  180 DELP=DPDU*(UNF-UNFA)/2.D0
      PNF=PNF+DELP
      UNF=UNFA+DELP/DPDU
      GO TO 60
C
CCC   VALUES CONVERGED
C
  190 U(KK)=UNF
      P(KK)=PNF
      T(KK)=TNF
      E(KK)=ENF
      BIT=XBF*RHONF
      RHO(KK)=RHONF
      GS1(KK,NF)=BIT
      GS2(KK,NF)=BIT*UNF
      GS3(KK,NF)=BIT*(ENF+UNF*UNF/2.D0/G)
      RETURN
C
CCC   ERROR MESSAGE
C
  200 WRITE(6,210)
  210 FORMAT(42H EXCESSIVE NUMBER OF ITERATIONS IN OUTFLO.)
      CALL EXIT
      END
```

```
      SUBROUTINE REFIT (NWAY,N)
C
C     ROUTINE TO PERFORM ALLOCATION OF MESH. CALLED AFTER EACH UPDATE
C     CYCLE BY BAR3 WITH NWAY = 1. ALSO CALLED INITIALLY BY BAR1 WITH
C     NWAY = 0.
C     ARGUMENT N IS A POINTER TO THE LEVEL AT WHICH THE
C     FLOW PARAMETERS ARE BEING INTERPOLATED.
C
C     REFIT ESTABLISHES MESH IN ACCORDANCE WITH VALUES OF INPUT
C     QUANTITIES DXMIN AND MAXDIM WHERE:
C
C          DXMIN - MINIMUM VALUE OF MESH INTERVAL (IN PHYSICAL UNITS)
C          MAXDIM - MAXIMUM NUMBER OF MESH POINTS DESIRED.
C
C     NDIM MESH POINTS ARE ESTABLISHED SUCH THAT 3.LE.NDIM.LE.MAXDIM.
C     AS THE PROPELLANT MOVES, POINTS ARE ADDED FROM TIME TO TIME AND
C     VALUES OF THE COMPUTATIONAL VARIABLES ARE DEDUCED BY A CUBIC
C     SPLINE INTERPOLATION.
C     IF THE INITIAL VALUE OF XB IS LESS THAN DXMIN, TERMINATION
C     OCCURS. IF XB IS GREATER THAN DXMIN BUT LESS THAN 2*DXMIN,
C     A WARNING IS PRINTED BUT EXECUTION CONTINUES.
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
C
      COMMON /BARA/ GS1(100,3), GS2(100,3), GS3(100,3)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARD/ DXMIN, DELTAX
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1  CV, ZR(20)
      COMMON /BARF/ XPR, VPR, CM, XB, VB, AB, PRM, RDOT, SIG, SSTART
      COMMON /BARG/ XBB,DELTBX,NPRC,NDIM1,NDIM2
      COMMON /BARJ/ QORF(100),PRPRES
      DIMENSION X(100,3,3), Y(100), SP(100), SPL(100)
      EQUIVALENCE (X(1,1,1),GS1(1,1))
C
      NDIMA=NDIM
      NDIM1A=NDIM1
      NDIM2A=NDIM2
      K=XB/DXMIN+0.5D0
      K=K+1
      K1=0
      IF(NPRC.EQ.0) GO TO 5
      IF(CM.GT.1.D-10) GO TO 4
      NPRC=0
      GO TO 5
    4 K1=XBB/DXMIN+0.5D0
      K1=K1+1
    5 IF (NWAY.EQ.1) GO TO 30
      SP(1)=1.D0
```

```
      DO 10 I=2,MAXDIM
   10 SP(I)=1.DO-0.062500/SP(I-1)
      IF (K.GE.3) GO TO 30
      IF (K.EQ.2) GO TO 20
      WRITE (6,120)
      CALL EXIT
   20 WRITE (6,130)
   30 K=MAXO(K,3)
      IF(NPRC.EQ.0) GO TO 34
      IF(K1.GE.3) GO TO 35
C
C     PROPELLING CHARGE TOO SHORT TO BE TREATED AS CONTINUUM.
C     DEFAULT TO LUMPED PARAMETER.
C
      NPRC=0
      RHOP=CM/AB/XB3
      PRPRES=QORF(NDIM1)
   34 NDIM=MINO(K,MAXDIM)
      GO TO 38
   35 IF(K+K1.GT.MAXDIM) GO TO 37
   36 NDIM=K
      NDIM1=K+1
      NDIM2=K+K1
      DELTBX=1.DO/(DFLOAT(K1)-1.DO)
      GO TO 38
   37 BIT=DFLOAT(MAXDIM)/DFLOAT(K+K1)
      K=DFLOAT(K)*BIT
      K=MAXO(K,3)
      K1=MAXDIM-K
      IF(K1.GE.3) GO TO 36
      K1=3
      K=MAXDIM-3
      GO TO 36
   38 DELTAX=1.DO/(DFLOAT(NDIM)-1.DO)
      IF (NWAY.EQ.0) RETURN
      IF (NDIM.EQ.NDIMA.AND.NPRC.EQ.0) RETURN
      IF(NDIM.EQ.NDIMA.AND.NDIM2.EQ.NDIM2A.AND.NPRC.EQ.1) RETURN
      DO 110 MM=1,3
      IL=2
      IR=NDIMA
      ILA=1
      IRA=NDIM
      MMM=NDIM
   39 BIT=1.DO/DFLOAT(IR-(IL-1))
      BITA=1.DO/DFLOAT(IRA-ILA)
      BITD=1.5DO/BIT/BIT
      IRM=IR-1
      IL1=IL
      ISHFT=IL-2
      IF (IL.GT.IRM) GO TO 60
```

```
      SPL(1)=0.D0
      DO 40 J=IL,IRM
      SPLB=BITD*(X(J+1,N,MM)+X(J-1,N,MM)-2.D0*X(J,N,MM))
      IF (J.GT.IL) SPLB=SPLB-0.25D0*SPL(J-1-ISHFT)/SP(J-2-ISHFT)
   40 SPL(J-ISHFT)=SPLB
      SPL(IR-ISHFT)=0.D0
      KSHFT=IRM+IL-ISHFT
      DO 50 J=IL,IRM
      JSHFT=KSHFT-J
   50 SPL(JSHFT)=(SPL(JSHFT)-0.25D0*SPL(JSHFT+1))/SP(JSHFT-1)
   60 DO 90 I=ILA,IRA
      ZA=BITA*DFLOAT(I-ILA)
      DO 70 J=IL1,IR
      Z=BIT*DFLOAT(J-(IL-1))
      IF (Z.GE.ZA) GO TO 80
   70 CONTINUE
      J=IR
   80 JJ=J-1
      IL1=J
      C1=(Z-ZA)/BIT
      C2=(ZA+BIT-Z)/BIT
      Y(I)=C1*X(JJ,N,MM)+C2*X(J,N,MM)
      IF (IL.GT.IRM) GO TO 90
      Y(I)=Y(I)-C1*C2/6.D0*BIT*BIT*((1.D0+C2)*SPL(JJ-ISHFT)+
     *(1.D0+C1)*SPL(J-ISHFT))
   90 CONTINUE
      IF(NPRC.EQ.0) GO TO 95
      IF(MMM.GT.NDIM) GO TO 95
      IL=NDIM1A+1
      IR=NDIM2A
      ILA=NDIM1
      IRA=NDIM2
      MMM=NDIM2
      GO TO 39
   95 DO 100 M=1,MMM
  100 X(M,N,MM)=Y(M)
  110 CONTINUE
      RETURN
C
  120 FORMAT (44H ILLEGAL VALUES OF XB AND DXMIN. TERMINATING)
  130 FORMAT (45H MARGINAL VALUES OF XB AND DXMIN. CONTINUING.)
      END
```

```
      FUNCTION RESP(X,V,A)
C
CCCCC
CCCCC RESISTIVE PRESSURE ACTING ON PROJECTILE AS A FUNCTION OF
CCCCC TRAVEL (X), VELOCITY (V). ARGUMENT A IS RETURNED IN CASES
CCCCC WHEN RESISTANCE IS PROPORTIONAL TO PROJECTILE ACCELERATION.
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
C
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1  CV, ZR(20)
      COMMON /BARH/ AUP,ADWN,VISLYR,DELYR,AMUV(10),AMU(10),DB,
     *              BRX(10),BR(10),AIRGAM,AIRPO,AIRTO,AIRMW,TWALO,
     *              PPPF,CDMUZ,ASBR,CDBR,PDIA,CHTW,DRHO(2)
      COMMON /BARH2/ PRMB,ELB,ANU,BMUV(10),BMU(10),NBRES3
      COMMON /BARH4/ RESAIR,RESOB
      COMMON /BARF/ XPR, VPR, CM, XB, VB, AB, PRM, RDOT, SIG, SSTART
      COMMON /BARI/ NWFR,NBRES1,NBRES2,NHTW,NMUZBL,NBRV
      DATA G/386.1600/,R/18550.00/
C
      RESP=0.00
      A=0.00
      IF(NBRES1.EQ.0) GO TO 40
CCC
CCC   RESISTANCE DUE TO FRICTION ON OBTURATOR GIVEN IN TABULAR FORM
CCC
      IF(X.GT.BRX(1)) GO TO 10
      RESP=BR(1)
      GO TO 40
   10 DO 20 I=2,NBRES1
      IF(BRX(I).GE.X) GO TO 30
   20 CONTINUE
      RESP=BR(NBRES1)
      GO TO 40
   30 BIT=(BRX(I)-X)/(BRX(I)-BRX(I-1))
      RESP=BR(I-1)*BIT+BR(I)*(1.00-BIT)
   40 RESOB=RESP
      IF(NBRES2.EQ.0) GO TO 60
CCC
CCC   RESISTANCE DUE TO SHOCK AHEAD OF PROJECTILE
CCC
   50 BIT=(AIRGAM-1.00)/(AIRGAM+1.00)
      BITC=DSQRT(G*AIRGAM*R*AIRTO/AIRMW)
      BITL=1.00-BIT
      BITU=(V+DSQRT(V*V+4.00*BITL*BITL*BITC*BITC))
     *     /(2.00*BITL*BITC)
      RESAIR=AIRPO*((1.00+BIT)*BITU*BITU-BIT)
      RESP=RESP+RESAIR
   60 IF(NBRES3.EQ.0) RETURN
C
```

```
C
C     RESISTANCE DUE TO FRICTION ON OBTURATOR AS DETERMINED
C     BY SETBACK PRESSURE AND TABULAR COEFFICIENT OF FRICTION
C
      IF(V.GT.BMUV(1)) GO TO 70
      BITMU=BMU(1)
      GO TO 100
   70 DO 80 I=2,NBRES3
      IF(BMUV(I).GE.V) GO TO 90
   80 CONTINUE
      BITMU=BMU(NBRES3)
      GO TO 100
   90 BIT=(BMUV(I)-V)/(BMUV(I)-BMUV(I-1))
      BITMU=BMU(I-1)*BIT+BMU(I)*(1.D0-BIT)
  100 RESOB=BITMU/BMU(1)*SSTART
      RESP=RESP+RESOB
      DENOM=(1.D0-ANU)/ANU*AB-BITMU*ELB*1.570796*DB
      IF(DENOM.LT.1.D-10) DENOM=1.D-10
      A=4.D0*BITMU*ELB/DB*PRMB/G/DENOM
      RETURN
      END
```

```
      FUNCTION RNOM(X)
C
C     NOMINAL DENSITY CURVE FOR PROPELLANT AS A FUNCTION OF
C     PRESSURE (X).
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1   CV, ZR(20)
      COMMON /BARH/ AUP,ADWN,VISLYR,DELYR,AMUV(10),AMU(10),
     *              BRX(10),BR(10),AIRGAM,AIRPO,AIRTO,AIRMW,TWALO,
     *              PBRF,CDMUZ,ASBR,CDBR,PDIA,CHTW,DRHO(2)
C
C
      DATA   G/386.16D0/
C
      RNOM=RHOP*(3.D0*G*X/AUP/AUP/RHOP+1.D0)**(1.D0/3.D0)
      RETURN
      END
```

```
      SUBROUTINE SIGCHK
C
CCC   CHECKS THAT PRESSURE IN SOLID PROPELLANT DOES NOT LIE
CCC   ABOVE NOMINAL LOADING CURVE.
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BARA/ GS1(100,3), GS2(100,3), GS3(100,3)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARG/ XBB,DELTBX,NPRC,NDIM1,NDIM2
C
      DO 10 I=NDIM1,NDIM2
      X=SNOM(GS1(I,NF))
      IF(GS3(I,NF).GT.X) GS3(I,NF)=X
   10 CONTINUE
      RETURN
      END
```

```
      FUNCTION SNOM(X)
C
CCC   NOMINAL STRESS CURVE FOR PROPELLANT AS A FUNCTION OF DENSITY
      (X).
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1   CV, ZR(20)
      COMMON /BARH/ AUP,ADWN,VISLYR,DELYR,AMUV(10),AMU(10),
     *              BRX(10),BR(10),AIRGAM,AIRPO,AIRTO,AIRMW,TWALO,
     *              PBRF,CDMUZ,ASBR,CDBR,PDIA,CHTW,DRHO(2)
C
      DATA G/386.1600/
C
      SNOM=AUP*AUP*RHOP/3.D0/G*((X/RHOP)**3-1.D0)
      RETURN
      END
```

```
      SUBROUTINE VELCHK
C
CCC   CHECKS THAT VELOCITY HAS NOT BEEN REVERSED AS A
CCC   CONSEQUENCE OF FRICTION
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON /BARA/ GS1(100,3), GS2(100,3), GS3(100,3)
      COMMON /BARC/ NDT, NI, NF, NP, INT, NDIM, MAXDIM
      COMMON /BARG/ XBB,DELTBX,NPRC,NDIM1,NDIM2
      COMMON /BARJ/ QORF(100),PRPRES
C
      NB=NI
      IF(INT.EQ.0) NB=NP
      DO 10 I=NDIM1,NDIM2
      IF(DABS(GS2(I,NB)).LT.1.D-10) GO TO 10
      IF(GS2(I,NF)*QORF(I).GT.0.D0) GS2(I,NF)=0.D0
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE WFR
C
CC    FRICTION BETWEEN SOLID PROPELLANT AND TUBE WALL
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MOL
C
      COMMON /BAR3/ RHO(100), P(100), E(100), T(100), U(100), ETA(100)
      COMMON /BARE/ GAM, BV, MOL, ECHEM, RHOP, B1(20), B2(20), BN(20),
     1  CV, ZR(20)
      COMMON /BARG/ XBP,DELTBX,NPRC,NDIM1,NDIM2
      COMMON /BARH/ AUP,ADWN,VISLYR,DELYR,AMUV(10),AMU(10),DB,
     *              BRX(10),BR(10),AIRGAM,AIRPO,AIRTO,AIRMW,TWALO,
     *              PBPF,CDMUZ,ASBR,CDBR,PDIA,CHTW,DRHO(2)
      COMMON /BARI/ NWFR,NBRES1,NPRES2,NHTW,NMUZBL,NBRV
      COMMON /BARJ/ QQRF(100),PRPRES
C
      IF(NWFR.GT.0) GO TO 20
C
CC    FRICTION DUE TO GAS FILM LAYER
C
      DO 10 I=NDIM1,NDIM2
      QQRF(I)=-4.D0/DB*VISLYR/DELYR*U(I)
   10 CONTINUE
      RETURN
C
CC    FRICTION DUE TO NORMAL STRESS
C
   20 DO 70 I=NDIM1,NDIM2
      QQRF(I)=0.D0
      IF(P(I).LE.0.D0) GO TO 70
      IF(I.EQ.NDIM1) GO TO 22
      IF(I.EQ.NDIM2) GO TO 24
      PATI=0.25D0*(P(I-1)+P(I+1)+2.D0*P(I))
      GO TO 26
   22 PATI=0.5D0*(P(I)+P(I+1))
      GO TO 26
   24 PATI=0.5D0*(P(I)+P(I-1))
   26 BITU=U(I)
      IF(BITU.GT.AMUV(1)) GO TO 30
      BITMU=AMU(1)
      GO TO 60
   30 DO 40 J=2,NWFR
      IF(AMUV(J).GE.BITU) GO TO 50
   40 CONTINUE
      BITMU=AMU(NWFR)
      GO TO 60
   50 BIT=(AMUV(J)-BITU)/(AMUV(J)-AMUV(J-1))
      BITMU=AMU(J-1)*BIT+AMU(J)*(1.D0-BIT)
   60 QQRF(I)=-4.D0/DB*BITMU*PATI
```

```
      IF(DABS(BITU).GT.1.D-10) QORF(I)=QORF(I)*BITU/DABS(BITU)
70    CONTINUE
      RETURN
      END
```

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